1998 Paper 5 Question 9

Foundations of Functional Programming

State the connection between the $\beta$-equality relation ($=_{\beta}$) and the $\beta$-reduction relation on $\lambda$-terms. Prove that $=_{\beta}$ is non-trivial, in the sense that there exist $\lambda$-terms $M$ and $N$ such that $M \neq_{\beta} N$. [4 marks]

Compare the call-by-name and call-by-value reduction strategies, giving examples to illustrate that

(a) sometimes the call-by-name strategy gives fewer reductions than the call-by-value strategy, and vice versa;

(b) the call-by-name strategy terminates when the normal form exists, whereas the call-by-value strategy need not. [6 marks]

Given the $\lambda$-term $(\lambda x.xI)(\lambda y.(\lambda z.\text{zzzz})(yt))$ where $I$ is $\lambda u.u$, display reduction paths arising from the call-by-name and call-by-value reduction strategies. Also, find the reduction path which consists of the fewest reduction steps and comment on your answer. [10 marks]