1998 Paper 1 Question 8

Discrete Mathematics

What is a partial order on a set? What is a total order?

Given two totally ordered sets $(A, \leqslant_A)$ and $(B, \leqslant_B)$, define the Product Order and Lexicographic Order on $A \times B$ and show that they are partial orders. Show that the Lexicographic Order is a total order and that it contains the Product Order. [10 marks]

Let S be the set of functions from $\mathbb{N}$ to $\{0, 1\}$. Define a relation F on S by

$$(f, g) \in F \iff \forall n \in \mathbb{N}.\ f(n) \leqslant g(n).$$

Show that F is a partial order.

Define a relation G on S by

$$(f, g) \in G \iff \sum_{n \in \mathbb{N}} f(n)10^{-n} \leqslant \sum_{n \in \mathbb{N}} g(n)10^{-n}.$$

Show that G is a total order and that it contains F. [10 marks]