1997 Paper 7 Question 10

Types

Let x range over a set of identifiers, i range over integer constants, r range over real constants and α range over a set of type variables. Now suppose we have a set of types, σ, given by

$$\sigma ::= \alpha \mid \mathit{int} \mid \mathit{real} \mid \sigma_1 \times \sigma_2 \mid \sigma_1 \to \sigma_2$$

and a language of terms, M, given by

$$M ::= x \mid i \mid r \mid \lambda x.M_1 \mid M_1 M_2 \mid (M_1, M_2).$$

Give ML-like type inference rules for formulae of the form $\Gamma \vdash M : \sigma$, explaining the form and rôle of Γ. [4 marks]

Explain the notion of *principal type* and state whether your set of rules has such a property. [4 marks]

Show from your rules that it is impossible to find a Γ and σ which enable inference of either $\Gamma \vdash \lambda f.(f(1), f(2.7)) : \sigma$ [2 marks]
or $\Gamma \vdash \lambda x.xx : \sigma$. [2 marks]

Now suppose we wish to do better than the usual ML treatment of overloading for operators like "+". So add to the language of types a conjunction connective

$$\sigma ::= \sigma_1 \wedge \sigma_2$$

where $M : \sigma_1 \wedge \sigma_2$ means informally that M has both types σ_1 and σ_2 and so can be used at either type. Add corresponding inference rules:

$$\frac{\Gamma \vdash M : \sigma \qquad \Gamma \vdash M : \sigma'}{\Gamma \vdash M : \sigma \wedge \sigma'}$$

$$\frac{\Gamma \vdash M : \sigma \wedge \sigma'}{\Gamma \vdash M : \sigma} \qquad\qquad \frac{\Gamma \vdash M : \sigma \wedge \sigma'}{\Gamma \vdash M : \sigma'}$$

Now show that a suitable σ_1 and σ_2 can be found such that

$$[\mathit{neg} : (\mathit{int} \to \mathit{int}) \wedge (\mathit{real} \to \mathit{real})] \vdash \lambda f.(f(\mathit{neg}(1)), f(\mathit{neg}(2.7))) : \sigma_1$$

and

$$[\,] \vdash \lambda x.xx : \sigma_2$$

hold. [8 marks]

[Hint: for the latter, you might give x a type $\sigma \wedge \sigma'$ where σ is a function type which accepts σ' as an argument.]