

1997 Paper 5 Question 10

Logic and Proof

Using binary predicate symbols $EQ (=)$ and $LT (<)$ and binary function symbols $SUM (+)$ and $PROD (\times)$, write down predicate calculus formulae that formalise the following statements (some of which are false) about the natural numbers:

(a) *there is a smallest number* [2 marks]

(b) *there is no largest number* [2 marks]

(c) *every number is the sum of two squares* [2 marks]

(d) *there exist two numbers whose product is less than their sum* [2 marks]

For each of the formulae (e) to (j) below, state whether it is valid (true in all interpretations) or not. Either give an informal justification of the validity, or outline a falsifying interpretation.

(e) $(\forall x P(x)) \rightarrow (\exists x P(x))$ [2 marks]

(f) $(\exists x P(x)) \rightarrow (\forall x P(x))$ [2 marks]

(g) $((\forall x P(x)) \wedge (\forall x Q(x))) \rightarrow (\forall x (P(x) \wedge Q(x)))$ [2 marks]

(h) $((\exists x P(x)) \wedge (\exists x Q(x))) \rightarrow (\exists x (P(x) \wedge Q(x)))$ [2 marks]

(i) $(\forall x \exists y P(x, y)) \rightarrow (\exists y \forall x P(x, y))$ [2 marks]

(j) $(\exists x \forall y P(x, y)) \rightarrow (\forall y \exists x P(x, y))$ [2 marks]