Numerical Analysis I

Explain the terms unit round off and machine epsilon (macheps). Why is machine epsilon used in preference to unit round off for practical purposes? [4 marks]

In the IEEE binary floating-point Standard (IEEE 754), what exponent and significand are used in representing each of the numbers 0, 1 and 2 in single precision? How are the exponent and significand stored in each case? [6 marks]

Show the 32 bits that represent (1+macheps). What is the exact value of macheps in this case? [4 marks]

What are the two sources of error in the formula

\[ f'(x) \approx \frac{f(x + h) - f(x)}{h} \]

and how does each type of error behave as \( h \) increases? [4 marks]

Suggest a suitable value of \( h \) if using this formula with IEEE single precision when \( f(x) = O(1) \). [2 marks]