# 1997 Paper 12 Question 13

## Numerical Analysis II

Define the *Chebyshev polynomial* $T_k(x)$. Evaluate $T_4(\frac{1}{2})$ using the formula $T_{k+1}(x) = 2xT_k(x) - T_{k-1}(x)$. What is the leading coefficient of $T_k(x)$? [4 marks]

The best $L_\infty$ approximation to $f(x) \in C[-1, 1]$ by a polynomial $p_{n-1}(x)$ of degree $n - 1$ has the property that

$$\max_{x \in [-1,1]} |e(x)|$$

is attained at $n + 1$ distinct points $-1 \leqslant \xi_0 < \xi_1 < \ldots < \xi_n \leqslant 1$ such that $e(\xi_j) = -e(\xi_{j-1})$ for $j = 1, 2, \ldots n$.

Let $f(x) = x^2$. Show, by means of a clearly labelled sketch graph, that the best polynomial approximation of degree 1 is a constant. [3 marks]

Now suppose $f(x) = x^3$ is the function to be approximated. Taking account of symmetry, sketch the graph of $f(x)$ and its best $L_\infty$ approximation by a polynomial of degree 2. [5 marks]

By differentiating $e(x)$, find the polynomial $p_2(x)$. [6 marks]

State a formula for the best approximation to $f(x) = x^n$ by a polynomial of degree $n - 1$. [2 marks]