1997 Paper 10 Question 9

Mathematics for Computation Theory

(*a*) Let A be a set of basic symbols, $\Phi = \{\phi_i\}_{i=1}^{k}$ be a finite set of operators, each of finite arity $r_i = ar(\phi_i)$. Define the language (A, Φ) of algebraic expressions over A. [4 marks]

State the Principle of Structural Induction for a predicate P defined on $L = (A, \Phi)$. [4 marks]

(*b*) Let $S = \{\vdash, X\}$ be an alphabet of two symbols. The language $L \subseteq S^*$ is defined as follows:

(*i*) $\vdash X \in L$;

(*ii*) if $\vdash y \in L$ for some string $y \in S^*$, then also $\vdash yy \in L$;

(*iii*) if $\vdash y \in L$ for some string $y \in S^*$, then also $\vdash yyy \in L$;

(*iv*) if $\vdash yXXXXX \in L$ for some string $y \in S^*$, then also $\vdash y \in L$;

(*v*) no other string $\in L$.

Determine whether the following strings belong to L:

$$(1) \quad \vdash XXXXXXX; \qquad\qquad (2) \quad \vdash XXXXX.$$

[Hint: it may be helpful to consider a predicate of the form "the number of Xs is not divisible by p" for a suitable prime p.] [12 marks]