1997 Paper 10 Question 11

Numerical Analysis I

Define *absolute error* and *relative error*. How are they related?   [2 marks]

Consider the quadratic expression $Q(x) = ax^2 + bx + c$ in which a, b, c and x are all represented with the same relative error δ.

In computing bx, estimate the worst-case relative error, and hence the worst-case absolute error.   [2 marks]

Now estimate the worst-case absolute error in computing $Q(x)$.   [4 marks]

Comment on the suitability of the formula

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

for computing one of the roots of $Q(x)$ in floating-point arithmetic. Derive an alternative formula and describe how it could be used in practice. Illustrate your answer by applying it to the case $a = 4$, $b = 400$, $c = 7$ on a decimal machine with only 4 significant digits available. [You may assume $\sqrt{b^2 - 4ac} \simeq b - (2ac/b)$ for the purposes of calculation.]   [12 marks]