

1997 Paper 10 Question 1

Continuous Mathematics

Suppose that some continuous function $f(x)$ has Fourier Transform $F(\mu)$. Now consider the consequences in the Fourier domain of each of the following operations upon $f(x)$:

(a) What will be the Fourier Transform of the n^{th} derivative of $f(x)$ with respect to x : $\frac{d^n}{dx^n} f(x)$? [4 marks]

(b) What will be the Fourier Transform after shifting $f(x)$ by a distance α : $f(x - \alpha)$? [4 marks]

(c) What will be the Fourier Transform after dilating $f(x)$ by a factor α : $f(x/\alpha)$? [4 marks]

(d) Suppose that $f(x)$ is convolved with another function $g(x)$ whose Fourier Transform is $G(\mu)$. What will be the Fourier Transform $H(\mu)$ of the convolution result $h(x)$ where

$$h(x) = \int_{-\infty}^{+\infty} f(\alpha)g(x - \alpha)d\alpha?$$

[4 marks]

(e) Suppose now that a two-dimensional continuous function $f(x, y)$ has a 2D Fourier Transform $F(\mu, \nu)$. Define the Laplacian operator ∇^2 and express the 2D Fourier Transform of $\nabla^2 f(x, y)$. [4 marks]