COMPUTER SCIENCE TRIPOS Part Ib

Tuesday 3 June 1997 1.30 to 4.30

Paper 4

Answer **five** *questions.*
Submit the answers in five **separate** *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* **all** *attempted questions, and circle the number of the question attached.*
Write on **one** *side of the paper only.*

1 Continuous Mathematics

Define Gabor wavelets in one dimension in three-parameter form, and explain the meaning of each of the three parameters. [5 marks]

Explain the Gabor–Heisenberg–Weyl "Uncertainty Principle". Express it as an inequality and define the quantities involved. Explain what is special about Gabor wavelets in terms of this principle. [5 marks]

If you construct an Information Diagram whose axes are time and frequency, how small an area in this plane can be occupied? Illustrate several differently parameterised Gabor wavelets in this plane, and explain what property they all share. How will they compare with all other possible functions in this plane? [5 marks]

Explain in what sense Gabor wavelets unify both the time domain and the frequency domain, constructing a continuous deformation between the two. Explain how the Fourier Transform is just a special case of the Gabor Transform. Discuss the cost implications of using the more general Gabor Transform in terms of the computational difficulty of obtaining the Gabor Transform coefficients. [5 marks]

2 Concurrent Systems

Explain the concept of serialisability of access to objects by concurrent processes. Consider the cases where operations that are meaningful to an application involve (a) only a single object and (b) related operations on a number of objects. Explore the cases where (c) the objects exist only in the main memory of a single computer for the duration of a program execution and (d) the objects exist in persistent store independent of program execution. [20 marks]

3 Further Modula-3

Describe the use of generic interfaces and implementations in Modula-3. [8 marks]

Why do generic modules have interfaces as parameters rather than types? [2 marks]

A Modula-3 library is required to model a set of values of some (generic) type. For illustration, a set could be modelled as an opaque object with methods to initialise it as an empty set, to insert a value of the element type, to test a value for membership in the set and to form a new set as the union with another set.

Write a generic interface for this set type. [5 marks]

Sketch an implementation including a concrete revelation of the type and signatures of the default methods but omitting their actual code. Indicate the kinds of constraints that might be imposed when instantiating the module. [5 marks]

4 Compiler Construction

Write brief notes on the possible target code sequences needed for the implementation of

(a) dynamically-sized local vectors

(b) labels and `GOTO` statements in a block-structured programming language

(c) access to free variables of functions using the static chain method

(d) an exception mechanism such as found in, for instance, ML or Modula-3

[5 marks each]

5 Data Structures and Algorithms

At frequent intervals 100,000 numbers arrive in a message. It is required to select and record their median and throw the rest away.

Describe and justify the algorithm you would use to do this as quickly as possible, and explain, as if to a manager, any risks you are taking. [6 marks]

If the manager insisted that the risk be either (*a*) very greatly reduced or even (*b*) eliminated, outline what you would do. [6 marks]

In case (*b*) explain to the manager why he is almost certainly misguided and then describe in detail the technique you would adopt. [8 marks]

6 Structured Hardware Design

Describe the behavioural constructs found in a Hardware Description Language (HDL) such as Verilog. [3 marks]

Give and briefly describe fragmentary examples of behavioural sections of an HDL which fall into the following categories:

(*a*) An example using only the RTL (register transfer language) subset of the language. [3 marks]

(*b*) An example using more than the RTL subset of the language, yet which is synthesisable. Is there a one-to-one correspondence between user-defined HDL variables and the resulting flip-flops? [3 marks]

(*c*) An example which is not typically synthesisable by today's general-purpose logic synthesisers. [3 marks]

Describe the use and benefits of a behavioural model for:

(*d*) rapid prototyping [2 marks]

(*e*) design entry [2 marks]

(*f*) simulation [2 marks]

(*g*) simulation test wrappers and stimulus generation [2 marks]

7 Operating System Functions

In the management of virtual memory, what is *thrashing*, and how does it occur? [5 marks]

What is the *working set* of a process, and how can it be computed? [5 marks]

List *five* techniques that can be used in an operating system to improve the performance of demand paged virtual memory. [5 marks]

What is a *capability*, and how can it be used for access control in a computer system? [5 marks]

8 Computation Theory

Define what is meant by saying that a set of *partial recursive* (μR) functions is *recursively enumerable*. Explain briefly how the universal register machine might be used to define a universal μR function $\mu(e, x)$ that enumerates the set of *all* partial recursive functions of a single variable x. [6 marks]

(a) Prove that the set of all total recursive functions of a single variable is not recursively enumerable. [4 marks]

(b) Show that there are recursively enumerable sets that are not recursive. [6 marks]

(c) Show that there is a partial recursive function that cannot be extended to any total recursive function. [4 marks]

[Any properties of recursively enumerable sets that you assume should be clearly stated.]

9 Numerical Analysis I

Explain the terms *unit round off* and *machine epsilon* (*macheps*). Why is *machine epsilon* used in preference to *unit round off* for practical purposes? [4 marks]

In the IEEE binary floating-point Standard (*IEEE* 754), what *exponent* and *significand* are used in representing each of the numbers 0, 1 and 2 in single precision? How are the exponent and significand stored in each case? [6 marks]

Show the 32 bits that represent (1+*macheps*). What is the *exact* value of *macheps* in this case? [4 marks]

What are the two sources of error in the formula

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h}$$

and how does each type of error behave as h increases? [4 marks]

Suggest a suitable value of h if using this formula with IEEE single precision when $f(x) = O(1)$. [2 marks]

10 Computer Graphics and Image Processing

Describe an algorithm to draw a straight line using only integer arithmetic. You may assume that the line is in the first octant, that the line starts and ends at integer coordinates, and that the function *setpixel*(x, y) turns on the pixel at location (x, y). [13 marks]

Explain how straight lines can be used to draw Bezier cubic curves. [7 marks]