Pi Calculus

What are the three kinds of commitment for a process in the π calculus? Explain informally how the commitments of $P | Q$ arise from those of $P$ or $Q$ or both.

We wish to implement a queue, to be weakly equivalent to the specification

$$Q\langle \rangle \overset{\text{def}}{=} \text{join}(x).Q\langle x \rangle$$

$$Q\langle x_1 \ldots x_n \rangle \overset{\text{def}}{=} \text{join}(x).Q\langle x_1 \ldots x_{n-1}x \rangle + \text{serve}(x_1).Q\langle x_2 \ldots x_n \rangle$$

The body of the queue is to consist of a chain of cells, each having the form $\pi(x, b)$; to hold a queue of $n$ items $x_1 \ldots x_n$ we define a chain parametrically on its head $a_0$ and tail $a_n$ as follows:

$$\text{Cells}(x_1 \ldots x_n) \overset{\text{def}}{=} (a_0a_n)(\nu a_1 \ldots a_{n-1})(\bar{a}_0(x_1, a_1) | \cdots | \bar{a}_{n-1}(x_n, a_n))$$

(a) Define agents $\text{Server}(a)$ and $\text{Joiner}(b)$ to manage the head and tail of the queue, respectively, with the intention that the system $\text{Queue}(x_1 \ldots x_n)$ defined below should be weakly equivalent to $Q\langle x_1 \ldots x_n \rangle$:

$$\text{Queue}\langle \rangle \overset{\text{def}}{=} (\nu a)(\text{Server}(a) | \text{Joiner}(a))$$

$$\text{Queue}(x_1 \ldots x_n) \overset{\text{def}}{=} (\nu ab)(\text{Server}(a) | \text{Cells}(x_1 \ldots x_n)(ab) | \text{Joiner}(b))$$

(b) In terms of your definition of $\text{Server}$ and $\text{Joiner}$, work out the commitments of $\text{Queue}(x_1 \ldots x_n)$ far enough to give an informal argument that it is indeed weakly equivalent to $Q\langle x_1 \ldots x_n \rangle$.

[Hint: treat the cases $n = 0$ and $n \neq 0$ separately.]