1996 Paper 6 Question 10

Logic and Proof

Briefly contrast the Davis–Putnam proof procedure with resolution. Illustrate your answer using proofs using both methods of

$$(P \to R) \wedge (\neg P \to \neg Q) \wedge (P \vee Q) \to (P \wedge R) \qquad \text{[8 marks]}$$

A polynomial over the integers, using modulo-2 arithmetic, can be regarded as a Boolean formula under the correspondence $1 \mapsto$ **true** and $0 \mapsto$ **false**. Show how to translate an arbitrary propositional formula to an equivalent polynomial, describing the translations of $\neg A$, $A \wedge B$, $A \vee B$, $A \to B$ and $A \leftrightarrow B$. [5 marks]

Use this translation to show that $(A \wedge B) \leftrightarrow (B \wedge A)$ is a tautology. [2 marks]

Use this translation to give a rule for simplifying formulæ of the form

$$(\ldots((A \leftrightarrow A) \leftrightarrow A)\ldots \leftrightarrow A) \qquad \text{[5 marks]}$$