1996 Paper 5 Question 9

Foundations of Functional Programming

Consider binary trees that are either empty (written **Lf**) or have the form **Br** $x\,t_1\,t_2$ where t_1 and t_2 are themselves binary trees. Give an encoding of binary trees in the λ-calculus, including functions **isLeaf**, **label**, **left** and **right** satisfying

$$\begin{aligned}
\textbf{isLeaf}\,\textbf{Lf} &\twoheadrightarrow \textbf{true}\\
\textbf{isLeaf}\,(\textbf{Br}\;x\,t_1\,t_2) &\twoheadrightarrow \textbf{false}\\
\textbf{label}\,(\textbf{Br}\;x\,t_1\,t_2) &\twoheadrightarrow x\\
\textbf{left}\,(\textbf{Br}\;x\,t_1\,t_2) &\twoheadrightarrow t_1\\
\textbf{right}\,(\textbf{Br}\;x\,t_1\,t_2) &\twoheadrightarrow t_2
\end{aligned}$$

If you use encodings of other data structures, state the properties assumed.

[6 marks]

Consider the ML functions f and g defined to satisfy

$$\begin{aligned}
f([], ys) &= ys\\
f(x :: xs, ys) &= f(xs, x :: ys)\\
g([], ys) &= ys\\
g(x :: xs, ys) &= x :: g(xs, ys)
\end{aligned}$$

Using list induction, prove $f(f(xs, []), []) = xs$. [14 marks]

[Hint: generalize this formula, making use of g. You may assume the equation $g(xs, []) = xs$.]