Compiler Construction

Describe a structure that could be used to represent the abstract syntax tree of the following λ-expression:

\[(\lambda a. (\lambda f. fa)(\lambda n.n + 1))3\]

Outline the definition of an evaluator function \(\text{eval}(\text{expr}, \text{env})\) that could be used to evaluate a given expression \(\text{expr}\) represented in this way in the context of an environment given by \(\text{env}\). Pay particular attention to the treatment of bound variables and the mechanism you use for function calls. [7 marks]

Is it possible with your implementation to give it a λ-expression which would cause \(\text{eval}\) to recurse to an unlimited depth? If so, give such a λ-expression; if not, explain why. [3 marks]

Is it possible with your implementation to give it a λ-expression that causes an environment chain of unlimited length to be created during the evaluation? If so, give a λ-expression that would cause this; if not, explain why it is not possible. [3 marks]

In an erroneous implementation of \(\text{eval}\) the value of

\[(\lambda x. \text{body})\]

is an object that does not incorporate the contextual environment, and whose call evaluates \(\text{body}\) in an environment derived from the environment of the call. Explain why such an implementation is wrong, giving an example λ-expression that would yield different results when evaluated by this implementation and yours. [7 marks]