

## 1996 Paper 4 Question 3

### Continuous Mathematics

Suppose we wish to compute a predicted value for some continuous, differentiable function  $f(t)$  at future time  $t = b$ , based only upon knowledge of its value now at time  $t = a$ , which is  $f(a)$ , and its first three derivatives now, namely  $f'(a)$ ,  $f''(a)$ , and  $f'''(a)$ . How would you compute such a prediction for the future value  $f(b)$ ? Give a formula or explain the sorts of terms that it would include and why they appear. [6 marks]

In practical computing problems with numerical data that are discretely sampled at periodic points in time, derivatives defined in terms of continuous limits must be replaced by discrete estimates. Provide finite-difference expressions for each of the three derivatives  $f'(t)$ ,  $f''(t)$ , and  $f'''(t)$  on the discrete set of samples

$$\dots, f(t_i), f(t_{i+1}), f(t_{i+2}), f(t_{i+3}), \dots$$

assuming that in this sequence the samples are separated by a constant interval  $\Delta t = t_{i+1} - t_i$ . [8 marks]

Real-world signals are generally accompanied by noise. Using the Differentiation Theorem of the Fourier Transform, comment upon how noise is amplified as a function of its spectral composition when computing first-order, second-order, and third-order derivatives of the signal. [6 marks]