1996 Paper 3 Question 3

Continuous Mathematics

We compute the representation of some continuous function $f(t)$ in a space spanned by an orthonormal family $\{\Psi_j(t)\}$ of continuous basis functions by projecting $f(t)$ onto them. We express these projections in bracket notation $<f(t), \Psi_j(t)>$ denoting $\int_{-\infty}^{\infty} f(t)\Psi_j(t)dt$, and $f(t)$ is assumed to be square-integrable.

(a) Give an expression for computing $f(t)$ if we know its projections $< f(t), \Psi_j(t) >$ onto this set of basis functions $\{\Psi_j(t)\}$. Explain what is happening. [5 marks]

(b) Now give an expression for computing $f^{(n)}(t)$, the nth derivative of $f(t)$ with respect to t, in terms of the same projections and continuous basis set. (You may assume the existence of all derivatives.) Explain your answer. [5 marks]

(c) Now consider a linear, time-invariant system with impulse-response function $h(t)$, having time-varying input $s(t)$ and time-varying output $r(t)$:

$$s(t) \longrightarrow \boxed{h(t)} \longrightarrow r(t)$$

In the case that the input is the complex exponential $s(t) = \exp(i\mu_j t)$ (where $i = \sqrt{-1}$ and μ_j is a constant), what can you say about the output $r(t)$ of such a system? [5 marks]

(d) If the input $s(t)$ has been represented in terms of a set of complex exponentials $\Psi_j(t) = \exp(i\mu_j t)$ as described at the beginning of this question, is it possible for *different* complex exponentials (not included in this set) to appear in the output $r(t)$ when it too is represented in terms of complex exponentials? Justify your answer. [5 marks]