1996 Paper 2 Question 1

Twenty-part question (*One mark per part*)

(*a*) Solve the difference equation $u_{n+1} = 2u_n + 1$ given that $u_0 = 0$.

(*b*) Solve the recurrence

$$f(0) = 0$$
$$f(1) = 1$$
$$f(n+2) = 7f(n+1) - 12f(n)$$

(*c*) Let A be a set. Given R, a relation on A, write $s(R)$ for its symmetric closure and $t(R)$ for its transitive closure. Give a set A and two relations R_1 and R_2 on A where R_1 satisfies
$$t(s(R)) = s(t(R))$$
and R_2 does not.

(d) Let S be a five-element set. Give the number of ways of picking 3 elements (not counting order) from S

   (i)   with replacement, and

   (ii) without replacement.

   Give the number of ways of picking 8 elements (not counting order) from S

   (iii) with replacement, and

   (iv) without replacement.

(e) Write a Modula-3 program using `IO.GetChar`, `Text.FromChar` and `IO.Put` to copy its standard input to its standard output.

(f) Write a Modula-3 procedure to calculate the greatest common divisor of two natural numbers.

(g) Given a character value, write a Boolean expression in Modula-3 that is true if the character is alphabetic or numeric.

(h) Given `fun d f x = f(f(x))` what is the type of `d`?

(i) Describe in words the strings represented by the regular expression $(aa^*b)^*a^*$.

(j) State the Pumping Lemma for regular languages.

(k) Give a regular grammar that generates the language consisting of even length strings of symbols from the alphabet $\{a, b, c\}$.

(l) Is the person best qualified to test a software component its implementor? Why or why not?

(m) Why is reliability harder to achieve in software than in other forms of engineering?

(n) Describe two of the three computer activities that were made criminal by the Computer Misuse Act of 1990.

(o) If you had a friend visiting from Southampton University who wanted to send an e-mail message to her boy-friend in Southampton, would you be breaking Information Technology Syndicate Rules to let her do this on your PWF account?

(*p*) You type the following command at a Unix shell prompt:

```
rm *
```

The `rm` program will not generally know that you typed a '`*`'. Why not?

(*q*) A lecturer wishes to carry out a secure on-line test of a class. Students run the test program while in a specified directory and by a specified deadline. The test program writes each student's score into a file. Students cannot read or write this file. Explain the Unix mechanism that supports these requirements.

(*r*) Given a fair die, show whether or not $\{1, 3, 5\}$ and $\{1, 4\}$ are independent events.

(*s*) Given $\mathrm{E}(X) = \mathrm{E}(X^2) = \frac{2}{3}$, evaluate $\mathrm{V}(X)$.

(*t*) In an M/M/1 queueing system, how is response time bounded as utilisation approaches unity?