1996 Paper 1 Question 1

Discrete Mathematics

Show that the number of undirected bi-partite graphs which have n vertices is

$$\sum_{k=0}^{n} \binom{n}{k} 2^{k(n-k)}.$$

For the purposes of this question regard such an undirected bi-partite graph as a triple (V, W, E) having disjoint sets of vertices V and W (with $|V \cup W| = n$) and edges $E \subseteq (V \times W)$. Note this means that the two graphs $(\{1,2\},\{3\},\{\})$ and $(\{1\},\{2,3\},\{\})$ are counted separately (because their partition differs) whereas one would more commonly argue that they are the same graph.

[10 marks]