

# 1996 Paper 11 Question 10

## Numerical Analysis I

Let  $x^*$  be the floating-point representation of a number  $x$ . Define the *absolute error* and *relative error* in representing  $x$  by  $x^*$ . How are these errors related? [3 marks]

Let  $x_1, x_2$  be two numbers. Find expressions for

(a) the *absolute error* in representing  $x_1 + x_2$

(b) the *relative error* in representing  $x_1.x_2$  (where “.” denotes multiplication) [4 marks]

Assume that the numbers 1 and 2 are represented exactly. Find an expression for the absolute error in calculating  $2x + 1$ . [2 marks]

In an iterative calculation the number  $y$  is an improved value of  $x$ , derived from the assignments

$$p := x/2 + 1$$

$$q := x - 2$$

$$y := p + 1/q$$

If  $\varepsilon_x$  is the absolute error in representing  $x$ , find an expression for the *absolute error*  $\varepsilon_y$  in representing  $y$ . [6 marks]

What is the approximate *relative error*  $\delta_y$  in representing  $y$  when  $x = 2.01$ ? [5 marks]