1 Concurrent Systems

Give a detailed criticism of the following options for supporting interactions between processes in separate address spaces:

(a) pipes

(b) asynchronous message passing

(c) synchronous message passing

[20 marks]

2 Further Modula-3

What are UNSAFE interfaces and implementations in Modula-3 and what are they used for? [5 marks]

Write brief notes on the following Modula-3 keywords:

UNTRACED
ADDRESS
ADR
LOOPHOLE
DISPOSE

[3 marks each]
3 Continuous Mathematics

Suppose we wish to compute a predicted value for some continuous, differentiable function $f(t)$ at future time $t = b$, based only upon knowledge of its value now at time $t = a$, which is $f(a)$, and its first three derivatives now, namely $f'(a)$, $f''(a)$, and $f'''(a)$. How would you compute such a prediction for the future value $f(b)$? Give a formula or explain the sorts of terms that it would include and why they appear. [6 marks]

In practical computing problems with numerical data that are discretely sampled at periodic points in time, derivatives defined in terms of continuous limits must be replaced by discrete estimates. Provide finite-difference expressions for each of the three derivatives $f'(t)$, $f''(t)$, and $f'''(t)$ on the discrete set of samples

$$\ldots, f(t_i), f(t_{i+1}), f(t_{i+2}), f(t_{i+3}), \ldots$$

assuming that in this sequence the samples are separated by a constant interval $\Delta t = t_{i+1} - t_i$. [8 marks]

Real-world signals are generally accompanied by noise. Using the Differentiation Theorem of the Fourier Transform, comment upon how noise is amplified as a function of its spectral composition when computing first-order, second-order, and third-order derivatives of the signal. [6 marks]
4 Compiler Construction

Describe a structure that could be used to represent the abstract syntax tree of the following λ-expression:

\((\lambda a. (\lambda f. f a)(\lambda n. n + 1))3\)

Outline the definition of an evaluator function \(\text{eval}(\text{expr}, \text{env})\) that could be used to evaluate a given expression \(\text{expr}\) represented in this way in the context of an environment given by \(\text{env}\). Pay particular attention to the treatment of bound variables and the mechanism you use for function calls. [7 marks]

Is it possible with your implementation to give it a λ-expression which would cause \(\text{eval}\) to recurse to an unlimited depth? If so, give such a λ-expression; if not, explain why. [3 marks]

Is it possible with your implementation to give it a λ-expression that causes an environment chain of unlimited length to be created during the evaluation? If so, give a λ-expression that would cause this; if not, explain why it is not possible. [3 marks]

In an erroneous implementation of \(\text{eval}\) the value of

\((\lambda x. \text{body})\)

is an object that does not incorporate the contextual environment, and whose call evaluates \(\text{body}\) in an environment derived from the environment of the call. Explain why such an implementation is wrong, giving an example λ-expression that would yield different results when evaluated by this implementation and yours. [7 marks]

5 Data Structures and Algorithms

Show why comparison-based sorting of \(n\) items cannot take much less than \(n \log n\) comparisons, being clear about your assumptions. Why can it take any less than \(n \log n\)? [10 marks]

If 1024 numbers are drawn randomly in the range 0–127 and sorted by binary insertion, about how many compares would you expect? A fairly rough estimate will do if your reasoning is clear. [10 marks]
6 Structured Hardware Design

Describe logic synthesis by writing short notes on each of the following topics:

(a) input language constructs
(b) output format
(c) use of libraries of subcircuits
(d) technology independence
(e) guiding metrics and constraints
(f) synthesis for multiple instances
(g) disadvantages compared with alternative approaches

[20 marks]

7 Operating System Functions

What is meant by the term demand paging in a virtual memory management system, and how is it implemented? [5 marks]

Briefly describe five techniques which the operating system and/or hardware can implement to improve the efficiency of demand paging. [5 marks]

What is the working set of a program, and how can an operating system use it in the management of virtual memory? [3 marks]

Describe the clock (second chance) algorithm for selecting a VM page for replacement when a page fault occurs. How is the performance of this algorithm affected by the memory size of the computer system, and how may this be avoided? [7 marks]
8 Computation Theory

A bag $B$ of natural numbers is a total function $f_B : \mathbb{N} \rightarrow \mathbb{N}$ giving for each natural number $x$ the count $f_B(x)$ of occurrences of $x$ in $B$. If each $f_B(x) = 0$ or 1, then $f_B$ is the characteristic function $\chi_S$ of a set $S$: every set can thus be regarded as a bag.

(a) A bag $B$ is recursive if the function $f_B$ is computable. Suppose that the sequence of bags $\{B_n \mid n \in \mathbb{N}\}$ is recursively enumerated by the computable function $e(n, x) = f_n(x)$, which gives the count of $x$ in each bag $B_n$. Show that there is a recursive set $S$ that is different from each bag $B_n$. [7 marks]

Hence prove that the set of all recursive bags cannot be recursively enumerated. [3 marks]

(b) A bag $B$ is finite if there is $X \in \mathbb{N}$ such that $f_B(x) = 0$ for all $x \geq X$. Show that the set of all finite bags is recursively enumerable. [10 marks]

9 Numerical Analysis I

Let $x^*$ be the floating-point representation of a number $x$. Define the absolute error and relative error in representing $x$ by $x^*$. How are these errors related? [3 marks]

Let $x_1$, $x_2$ be two numbers. Find expressions for

(a) the absolute error in representing $x_1 + x_2$

(b) the relative error in representing $x_1 \cdot x_2$ (where "\cdot" denotes multiplication) [4 marks]

Assume that the numbers 1 and 2 are represented exactly. Find an expression for the absolute error in calculating $2x + 1$. [2 marks]

In an iterative calculation the number $y$ is an improved value of $x$, derived from the assignments

\[
p := x/2 + 1 \\
q := x - 2 \\
y := p + 1/q
\]

If $\varepsilon_x$ is the absolute error in representing $x$, find an expression for the absolute error $\varepsilon_y$ in representing $y$. [6 marks]

What is the approximate relative error $\delta_y$ in representing $y$ when $x = 2.01$? [5 marks]
10 Graphics

Describe a quad-tree encoding method for greyscale images. [6 marks]

Given the following greyscale image, draw a diagram showing how it would be encoded using your method from the previous part.

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[4 marks]

An image processing package allows the user to design $3 \times 3$ convolution filters. Design $3 \times 3$ filters to perform the following tasks:

(a) blurring [2 marks]

(b) edge detection of vertical edges [2 marks]

Choose one of the two filters (a) or (b) from the previous part. Explain how it works, using the following image as an example (you may round off any calculated values to the nearest integer).

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[6 marks]