COMPUTER SCIENCE TRIPOS Part IA

Tuesday 4 June 1996 1.30 to 4.30

Paper 2

Answer the question in Section A, **one** *question from each of Sections B and C, and* **two** *questions from Section D.*

Submit the answers in five separate *bundles, each with its own cover sheet. On each cover sheet, write the numbers of* all *attempted questions, and circle the number of the question attached.*

Write on **one** *side of the paper only.*

SECTION A

1 Twenty-part question (*One mark per part*)

(*a*) Solve the difference equation $u_{n+1} = 2u_n + 1$ given that $u_0 = 0$.

(*b*) Solve the recurrence

$$\begin{aligned} f(0) &= 0 \\ f(1) &= 1 \\ f(n+2) &= 7f(n+1) - 12f(n) \end{aligned}$$

(*c*) Let A be a set. Given R, a relation on A, write $s(R)$ for its symmetric closure and $t(R)$ for its transitive closure. Give a set A and two relations R_1 and R_2 on A where R_1 satisfies

$$t(s(R)) = s(t(R))$$

and R_2 does not.

(d) Let S be a five-element set. Give the number of ways of picking 3 elements (not counting order) from S

(i) with replacement, and

(ii) without replacement.

Give the number of ways of picking 8 elements (not counting order) from S

(iii) with replacement, and

(iv) without replacement.

(e) Write a Modula-3 program using `IO.GetChar`, `Text.FromChar` and `IO.Put` to copy its standard input to its standard output.

(f) Write a Modula-3 procedure to calculate the greatest common divisor of two natural numbers.

(g) Given a character value, write a Boolean expression in Modula-3 that is true if the character is alphabetic or numeric.

(h) Given `fun d f x = f(f(x))` what is the type of `d`?

(i) Describe in words the strings represented by the regular expression $(aa^*b)^*a^*$.

(j) State the Pumping Lemma for regular languages.

(k) Give a regular grammar that generates the language consisting of even length strings of symbols from the alphabet $\{a, b, c\}$.

(l) Is the person best qualified to test a software component its implementor? Why or why not?

(m) Why is reliability harder to achieve in software than in other forms of engineering?

(n) Describe two of the three computer activities that were made criminal by the Computer Misuse Act of 1990.

(o) If you had a friend visiting from Southampton University who wanted to send an e-mail message to her boy-friend in Southampton, would you be breaking Information Technology Syndicate Rules to let her do this on your PWF account?

(p) You type the following command at a Unix shell prompt:

```
rm *
```

The `rm` program will not generally know that you typed a `'*'`. Why not?

(q) A lecturer wishes to carry out a secure on-line test of a class. Students run the test program while in a specified directory and by a specified deadline. The test program writes each student's score into a file. Students cannot read or write this file. Explain the Unix mechanism that supports these requirements.

(r) Given a fair die, show whether or not $\{1, 3, 5\}$ and $\{1, 4\}$ are independent events.

(s) Given $\mathrm{E}(X) = \mathrm{E}(X^2) = \frac{2}{3}$, evaluate $\mathrm{V}(X)$.

(t) In an M/M/1 queueing system, how is response time bounded as utilisation approaches unity?

SECTION B

2 Digital Electronics

(a) What is the purpose of a tri-state buffer? Give an example of how it might be used. [5 marks]

(b) Find the simplest sum-of-products form for the function f with the don't care condition g, where

$$f = a(b\overline{c} + bc + \overline{b}\overline{c}d) + b\overline{d}(\overline{c} + a)$$

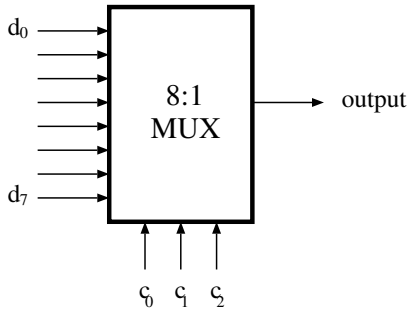
$$g = a\overline{b}(cd + \overline{c}\overline{d}) + \overline{a}\,\overline{c}d$$

[6 marks]

(c) A Gray code is a sequence of codes which differ in one bit position at each step. For example $00, 01, 11, 10, 00, ...$ is a two-bit Gray code. Design a counter made from JK-flipflops to produce a three-bit Gray code. [9 marks]

3 Digital Electronics

(a) A multiplexer is a device that selects one of its inputs as the output. The selection is determined by a set of control signals. For example, in the 8:1 multiplexer shown below, the output will be equal to d_6 when $c_2 = 1$, $c_1 = 1$ and $c_0 = 0$.



Give a circuit which implements this 8:1 multiplexer using only NAND gates. [10 marks]

(b) Using only 8:1 multiplexers, show how to build a 16:1 multiplexer. [4 marks]

(c) Show how an 8:1 multiplexer and a single inverter can be used to implement any combinational function of four variables. (You may assume the availability of signals for logical 1 and logical 0.) [6 marks]

SECTION C

4 Probability

What is a *probability generating function*? [4 marks]

If a random variable X is distributed Geometric(p) then $\mathrm{P}(X = r) = (1-p)^r p$. Derive a probability generating function which is appropriate for the Geometric distribution. [4 marks]

Determine the expectation and variance of X. [3+3 marks]

A game which uses a fair die requires each player to throw a six to start. What is the number of times that a player may expect to thow the die before achieving the required six? [4 marks]

What is the variance of this number? [2 marks]

5 Probability

If n coins are tossed, the number of ways in which r can land heads is ${}^nC_r$. Given that $n, r \in \mathbb{N}$ and that $0 \leqslant r \leqslant n$, Pascal's Theorem states:

$${}^nC_r = \begin{cases} 1, & \text{if } r = 0 \text{ or } r = n \\ {}^{n-1}C_{r-1} + {}^{n-1}C_r, & \text{otherwise} \end{cases}$$

Prove Pascal's Theorem. [6 marks]

Hence prove that:

$${}^nC_r = \frac{n!}{(n-r)!\,r!}$$

[7 marks]

Show that:

$$\sum_{r=0}^{n} {}^nC_r = 2^n$$

[7 marks]

SECTION D

6 Professional Practice and Ethics

What is the purpose of professional societies such as the British Computer Society? [20 marks]

7 Software Engineering

Describe the process of specifying a major piece of software: the main documents produced, their immediate purpose and their ongoing role in the software life cycle. [10 marks]

Describe the role that formal methods can play at each stage of the software life cycle. Explain any disadvantages of the uses of formal methods that you have discussed. [10 marks]

8 Regular Languages and Finite Automata

Show that if L is a regular language then the set of strings in L of odd length is also a regular language. Is the same true of strings of even length? Justify your answer. [8 marks]

If L is regular language let L' be the set of strings in L that are palindromes. Is is possible that L' is regular? Will L' necessarily be regular? Explain your answer with suitable examples and proofs. [6 marks]

It is known that the language *Pal* consisting of all palindromes is not regular. If possible find a regular language L such that L is a subset of *Pal*, or if this is not possible explain why. Similarly either find a regular language L' so that *Pal* is a subset of L', or again explain why this can not be done. [6 marks]

9 Unix Case Study

(a) Draw a diagram of the address space of a Unix process. Indicate which parts may be shared with other processes. [5 marks]

(b) Explain the Unix system call mechanism. [5 marks]

(c) Describe how the fork system call is implemented. Discuss the mechanism from the viewpoint of efficiency and support for sharing between families of processes. How may a parent synchronise with the termination of a child? [6 marks]

(d) Contrast the execution by the shell of the commands

```
c1 > file1
c2 < file1 > file2
c3 < file2
```

with the execution of the single command

```
c1 | c2 | c3
```

[4 marks]