

COMPUTER SCIENCE TRIPOS Part IA

Monday 3 June 1996 1.30 to 4.30

Paper 1

Answer **two** questions from Section A, and **one** question from **each** of sections B, C, D and E.

Submit the answers in six separate bundles, each with its own cover sheet. On each cover sheet, write the numbers of all attempted questions, and circle the number of the question attached.

Write on **one** side of the paper only.

SECTION A

1 Discrete Mathematics

Show that the number of undirected bi-partite graphs which have n vertices is

$$\sum_{k=0}^n \binom{n}{k} 2^{k(n-k)}.$$

For the purposes of this question regard such an undirected bi-partite graph as a triple (V, W, E) having disjoint sets of vertices V and W (with $|V \cup W| = n$) and edges $E \subseteq (V \times W)$. Note this means that the two graphs $(\{1, 2\}, \{3\}, \{\})$ and $(\{1\}, \{2, 3\}, \{\})$ are counted separately (because their partition differs) whereas one would more commonly argue that they are the same graph.

[10 marks]

2 Foundations of Computer Science

The ML data type `BOOL`, defined below, is to be used to represent boolean expressions.

```
datatype BOOL = VAR of string
              | NOT of BOOL
              | AND of BOOL*BOOL
              | OR  of BOOL*BOOL;
```

The constructor `VAR` is used to represent named boolean variables, and `NOT`, `AND` and `OR` are used to represent the corresponding boolean operators.

Define a function that will return a list of the distinct names used in a given boolean expression. [4 marks]

A context is represented by a list of strings corresponding to the boolean variables that are set to true. All other variables are deemed to be set to false. Define a function that will evaluate a given boolean expression in a given context. [3 marks]

Incorporate your two functions into a program that will determine whether a given boolean expression is true for all possible settings of its variables. [3 marks]

3 Programming in Modula-3

Show how Modula-3 exceptions can be used in conjunction with a `FOR` loop

(a) to simulate the effect of the `EXIT` statement; [5 marks]

(b) to skip to the next iteration of the loop. [5 marks]

4 System Design

(a) What are the advantages of using a high-level language for writing programs? [7 marks]

(b) Why must some parts of the code which makes up an executable binary program typically have to be written in assembly language? Who typically writes these parts and where are they normally stored in the system? [3 marks]

SECTION B

5 Foundations of Computer Science

Define an ML function `rotations` that will compute the list of all rotations of a given list. For example

```
rotations [1,2,3] = [[1,2,3], [2,3,1], [3,1,2]]
```

The order in which the rotations occur is unimportant. [10 marks]

Carefully explain how your function works and estimate the time complexity of your solution. [10 marks]

6 Foundations of Computer Science

Give the declaration of an ML datatype that could be used in the representation of a lazy list of integers, and illustrate its use by defining a function `ints` that when given an argument `n` yields a lazy list of integers from `n` to infinity. [5 marks]

The decimal representation of a real number in the range 0 to 1 is to be represented as an infinite sequence of the decimal digits following the decimal point ($0.d_1d_2\dots$). Define a function `mknumb` which when applied to the digit function `dig` will construct a lazy list of these digits where the i^{th} digit (d_i) is given by `dig i`. [5 marks]

Suppose we have an infinite sequence of such numbers $[r_1, r_2, \dots]$, in which the digits of the decimal expansion of r_i are given by the digit function f_i , and that the collection of digit functions is represented by the lazy list $[f_1, f_2, \dots]$. Define suitable datatypes for the list of numbers and the list of digit functions. [5 marks]

Define a function `newnumb` which when given the lazy list of digit functions will yield a lazy list of digits that have the property that its i^{th} digit differs from the i^{th} digit of r_i . [5 marks]

SECTION C**7 Discrete Mathematics**

State the requirements for (S, \leq) to be a partially-ordered set. Define the notion of a *topological sort* of S . [10 marks]

Let (S, \leq_S) and (T, \leq_T) be partially ordered sets. We say that $f : S \rightarrow T$ is an *isomorphism* of (S, \leq_S) and (T, \leq_T) if f is a bijection and it preserves the order i.e.

$$(\forall x, y \in S)(x \leq_S y \Leftrightarrow f(x) \leq_T f(y)).$$

If such an isomorphism exists we say that (S, \leq_S) and (T, \leq_T) are *isomorphic*.

Give two topological sorts of $\mathbb{N} \times \mathbb{N}$ ordered by the product order, one of which is isomorphic to \mathbb{N} with the usual ordering and the other is not. [10 marks]

8 Discrete Mathematics

The fiercely logical inhabitants of planet Volcano use coins whose values are all powers of two, the smallest being the one-pfatz (1pf) coin. Designs for all non-negative integer powers of two exist, but in practice computers are used instead of physically minting very high value coins.

For entry to the moon-fleet academy a popular question is to enquire as to the number of different ways of representing sums of money (ignoring order of course) of the form npf . Show that the number of ways $w(n, k)$ of representing npf using coins of value up to $2^k pf$ satisfies the recurrence

$$w(n, k) = w(n - 2^k, k) + w(n, k - 1)$$

and add appropriate base case(s). [5 marks]

Show, assuming n is a multiple of 4, that

$$w(n, 2) = (n/4 + 1)^2.$$

[5 marks]

Show also, if n is a multiple of 2^k with $k > 0$, that

$$w(n, k) \leq \frac{n}{2^k} w(n, k - 1) + 1.$$

[5 marks]

Using the above or otherwise show further that

$$2^k \leq w(2^k, k) \leq 2^{k^2}.$$

[Hint: for the lower bound you might well consider how one can derive one way of representing 2^{k+1} from each way of representing 2^k but only by using coins of even value and use induction.] [5 marks]

SECTION D

9 Programming in Modula-3

A student unfamiliar with Modula-3 wrote the following procedure:

```

PROCEDURE Stats () =
  VAR m, v := 0.0;
  BEGIN
    WITH n = IO.GetInt (),
         a = ARRAY [1..n] OF REAL DO
      FOR i := 1 TO n DO a[i] := IO.GetReal () END;
      FOR i := 1 TO n DO INC (m, a[i]) END;
      m := m / n;
      FOR i := 1 TO n DO INC (v, (a[i]-m)*(a[i]-m)) END;
      v := v / n;
      RETURN RECORD m, v;
    END;
  END Stats;

```

Explain, to the extent that you can reconstruct them, the programmer's intentions. [5 marks]

Comment on the legality of the code and indicate what the Modula-3 compiler's response would be to it. [5 marks]

How would you correct the procedure –

- (a) following the external data format and internal data structure and algorithm intended by the programmer; [5 marks]
- (b) re-writing it completely from scratch, possibly even having a different external data format (which should be specified). [5 marks]

10 Programming in Modula-3

A priority queue maintains a list of pairs, each consisting of a priority (a natural number) and a name (a text string), sorted into increasing order of priority. Consider the implementation of such a queue as an object in Modula-3. This should have two methods — **insert** taking a pair as its arguments to be placed at the appropriate point in the list, and **next** taking no arguments but removing the first pair from the list and returning its text string component.

Give type definitions and default method implementations for this queue object. [20 marks]

SECTION E

11 System Design

- (a) How does a loaded program in a general purpose timesharing computer gain access to input and output streams provided by the operating system? [5 marks]
- (b) What happens when the program is ready to receive the next data from a keyboard input stream and no key press is ready? [5 marks]
- (c) A CPU intensive program will tend to run for long periods of time without making requests of the operating system. What happens to other users programs when someone runs a CPU intensive program? [5 marks]
- (d) What can be done for a shared output device such as a printer or plotter to ensure civilised results when two or more users try to use it at once? [5 marks]

12 System Design

With modern technology, many pieces of equipment which we would describe as a ‘computer’ actually contain multiple hardware components, each performing fetch-execute cycles. However not all of these are always intended to be ‘seen’ by the programmer as part of the programming model. Define or explain the following:

- (a) The term *microprocessor*. [3 marks]
- (b) The term *programming model*. [3 marks]
- (c) The effects (or lack of them) on the programming model of having microprocessors imbedded in peripheral devices of a computer. [6 marks]
- (d) A programming model of an overt multiprocessor machine. [6 marks]
- (e) What type of system in production today is likely to have exactly one microprocessor inside? [2 marks]