1995 Paper 7 Question 11

Specification and Verification II

Describe the choice operator, ε, of higher-order logic, giving examples of its use. [6 marks]

Using `First` and `Next`, where

```
First p t = (∀t'. t' < t ⇒ ¬(p t')) ∧ p t
Next p (t1,t2) = (t1 < t2) ∧ (∀t.t1 < t ∧ t < t2 ⇒ ¬(p t)) ∧ p t2
```

define a function `TimeOf` such that for a function, `f`, `TimeOf f n` returns the time when `f` becomes true for the $\mathtt{n}^{th}$ time. [4 marks]

Explain the significance of the following general theorem for temporal abstraction. Instantiate `r` and `f` for the case of a positive edge-triggered D-type flipflop, and describe what the general theorem states for these instantiations.

```
⊢ ∀f r.
    (∃t. f t) ∧
    (∀t. f t ⇒ (∃n. Next f (t,t+n) ∧ r(t,t+n))) ⇒
      (∀u. r(TimeOf f u,TimeOf f (u + 1)))
```

[10 marks]