1995 Paper 12 Question 13

Complexity Theory

Comment on each of the following statements about Computational Complexity. Indicate any places where their wording is not sufficiently precise. For each, decide whether the statement is true, partially true, true but improperly justified, false or just muddled.

(a) Given a variant on Quicksort that uses a median-of-three procedure to select pivots, it seems hard to identify exactly what ordering of input data will make the quicksort behave worst. But because there are $N!$ possible different orderings and $N!$ is a bit like 2^N the problem is an NP one.

(b) If we could solve the Boolean Satisfiability problem efficiently we could use that to simulate the behaviour of *any* Turing machine and hence solve all other problems efficiently.

(c) To keep your secret treasure safe you intend to dig out a series of caves and tunnels forming a maze. You invent a graph for which you know a Hamiltonian circuit (for example, you start by putting in edges to make that circuit and then add lots more to make the graph more complicated). The Hamiltonian circuit problem is known to be NP complete, so given just the graph nobody except you will be able to find the Hamiltonian circuit. You wire up the tunnels so that the treasure can only be reached (safely!) by traversing the Hamiltonian circuit. Being arrogant you then pin details of the graph on your door. NP completeness means that your treasure is almost certainly secure.

You should provide a brief overview of any result, construction or proof that you refer to, but you are not expected to work through the details.

[20 marks]