State carefully the conditions for a relation $R$ on a set $S$ to be:

(a) a partial order;

(b) a total order;

(c) a well-order, as applied to both (a) and (b). [6 marks]

Let $S$ consist of ordered pairs of positive integers $(i, j)$ such that $j > i$. Define relations on $S$ that are

(a) totally and well-ordered;

(b) partially (not totally) and well-ordered;

(c) totally but not well-ordered. [14 marks]

In each case, explain why.