1994 Paper 10 Question 12

Introduction to Functional Programming

Recall that `f o g` is the function that maps `x` to `f(g(x))`. Consider the ML definitions

```
fun I x = x;
fun pair (f,g) (x,y) = (f x, g y);
fun pup (f,g) z = (f z, g z);
fun fst (x,y) = x;
fun snd (x,y) = y;
```

Describe the effect of the following functions:

```
pair(I,I)                pair(f1 o f2, g1 o g2)
pup(fst,snd)             pup(f o fst, g o snd)
```
[4 marks]

Infinite lists can be represented in a functional language by triples. A triple of the form (a, h, t) represents the infinite list whose nth element is $h(t^n(a))$ for $n \geqslant 0$.

(a) Give a representation for the infinite list $n, n+1, n+2, \ldots$ [2 marks]

(b) Code in ML a map functional for this representation; given a function f and the infinite list $x_0, x_1, \ldots$, it should yield the representation of $f(x_0), f(x_1), \ldots$ [3 marks]

(c) Code in ML a zip function, which combines the infinite lists $x_0, x_1, \ldots$ and $y_0, y_1, \ldots$ to the list of pairs $(x_0, y_0), (x_1, y_1), \ldots$ [4 marks]

(d) Code in ML an interleave function, which combines the infinite lists $x_0, x_1, \ldots$ and $y_0, y_1, \ldots$ to yield $x_0, y_0, x_1, y_1, \ldots$ [5 marks]

(e) How does this representation compare with the usual representation of infinite lists in ML? Briefly discuss. [2 marks]