COMPUTER SCIENCE TRIPOS Part Ib

Monday 30 May 1994 1.30 to 4.30

Paper 3

Answer **five** *questions.*
Submit the answers in five **separate** *bundles each with its own cover sheet.*
Write on **one** *side of the paper only.*

1 Programming Language Compilation

It is required to write a program to gather information about the lexical structure of a source file. The file contains words composed of letters and digits starting with a letter, decimal integers composed of digits, and various other single- and multi-character symbols such as `:=`, `+`, `-`, `[`, `]`, and `->`. About 50 of the words are reserved, the others being identifiers.

It is required to count how often each symbol and reserved word occurs, the number of distinct identifiers used, and the mean value of all the integers that occur in the source file. Suggest how you would design such a program with the aid of Lex, paying particular attention to the overall structure of the program, and giving a detailed account of how you would deal with the identifiers. [20 marks]

2 Further Modula-3

The thread system in Modula-3 uses *mutexes* and *condition variables* to control concurrency. An alternative scheme would be to provide *eventcounts* and *sequencers*. An eventcount is an integer, initially zero, equipped with the three atomic operations:

`advance` increments the count and returns its new value,

`read` returns the current value of the count, and

`await (value)` suspends the calling thread until the count is at least as large as the value given as an argument.

A sequencer is an integer, initially zero, equipped with a single atomic operation:

`ticket` increments the count and returns its previous value.

Given an eventcount, `guard`, and a sequencer, `turn`, a critical region can then be coded as follows:

```
myturn := turn.ticket ();
guard.await (myturn);
    .
    .    protected code
    .
EVAL guard.advance ();
```

Write an interface, `ECS`, defining opaque object types `EventCount` and `Sequencer`. `EventCount` should have methods `advance`, `read` and `await`, with appropriate signatures, and `Sequencer` should have a `ticket` method. [8 marks]

Sketch an implementation of the `ECS` module giving concrete revelations of the types and providing appropriate default methods. [12 marks]

3 Formal Languages and Automata

What is meant by the *language accepted* by a finite deterministic automaton $M = (Q, \Sigma, \delta, i, F)$? [2 marks]

Show that it is possible to associate with M a regular expression $\mathbf{r}$ over Σ denoting the same language as that accepted by M. [12 marks]

Illustrate your answer by constructing such a regular expression $\mathbf{r}$ when M is the finite deterministic automaton with

$$\begin{aligned} Q &= \{q_1, q_2, q_3\} \\ \Sigma &= \{0, 1\} \\ i &= q_1 \\ F &= \{q_3\} \end{aligned}$$

and with transition function δ defined by the table

	q_1	q_2	q_3
0	q_2	q_3	q_3
1	q_3	q_2	q_2

[6 marks]

4 Operating System Functions

Describe the use of an *inverted page table* for the implementation of virtual addressing and a paging virtual memory system. Give details of the operation of the page table and its associated data structures. [15 marks]

What can be done to overcome the thrashing problem encountered with a direct mapped inverted page table? [5 marks]

5 Operating System Functions

What is the *Access Control matrix*? Describe the *capability* and *access control list* views of this matrix. [10 marks]

In a capability-based system, describe the techniques which can be used to protect the capabilities from unauthorised modification. [10 marks]

6 Data Structures and Algorithms

For the following, n is a positive integer and G is a graph of N nodes (vertices) and E arcs (edges) each with a given weight (or cost). For *seven* of the following indicate, with a short justification, whether the statement is true or false.

(a) All functions f of the form $f(n) = An^k$ (with A and k being constants) are in the class $O(2^n)$.

(b) All sorting methods for an array of n elements take time $O(n^5)$.

(c) It is possible to sort an array of n elements using binary comparisons in $\Theta(n \log n)$ time.

(d) It is possible to sort an array of n elements using binary comparisons using $O(1)$ (i.e. constant independent of n) additional space.

(e) Radix sorting can sort any set of integers in linear time.

(f) All straight lines from the inside of a polygon to the outside intersect the points on the edges forming its boundary an odd number of times.

(g) It is always cheaper to find the shortest distance between two given nodes u, v of G than to find all N shortest distances from u to every other node.

(h) It is possible to find the shortest paths between all N^2 pairs of nodes of G in $O(N^3)$ time.

(i) If G is connected then the minimal spanning subtree of G contains the $N - 1$ edges whose weights are smallest.

(j) It is possible to compute the convex hull of n points in a plane in time $g(n)$ where g is a function such that $g(n)/n^2$ tends to zero as n becomes large.

Marks will be awarded for overall succinctness, attention to detail and absence of random guesses lacking justification.

[20 marks]

7 Data Structures and Algorithms

For *four* of the following ideas, give examples of problems and algorithms to solve them.

(a) divide and conquer

(b) breadth-first search

(c) balanced data structures

(d) use of randomness in algorithms

(e) "greedy" algorithms

(f) dynamic programming

[5 marks each]

8 Graphics

Explain with a diagram how a shadow mask cathode ray tube works. [12 marks]

What might be the point of extending the scheme to accommodate five electron guns? [8 marks]

9 Numerical Analysis I

The mid-point rule can be expressed in the form

$$I_n = \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} f(x)\,dx = f(n) + e_n$$

where

$$e_n = f''(\theta_n)/24$$

for some θ_n in the interval $(n - \frac{1}{2}, n + \frac{1}{2})$. Assuming that a formula for $\int f(x)\,dx$ is known, and using the notation

$$S_{p,q} = \sum_{n=p}^{q} f(n),$$

describe a method for estimating the sum of a slowly convergent series $S_{1,\infty}$, by summing only the first N terms and estimating the remainder by integration. [7 marks]

Assuming that $f''(x)$ is a positive decreasing function, derive an estimate of the error $|E_N|$ in the method. [5 marks]

Given

$$\int \frac{dx}{1+x^2} = \tan^{-1} x,$$

apply the method to

$$\sum_{n=1}^{\infty} \frac{1}{1+n^2}.$$

What is the integral remainder to be added to $S_{1,N}$? [4 marks]

To the nearest power of 10, how large should N be to achieve an absolute error of approximately 10^{-16}? [4 marks]

10 Structured Hardware Design

A subsystem consists of a microprocessor core, RAM, ROM and a sophisticated DMA engine (which effectively consists of counters and random logic), and is to be built into a wide variety of modules with various peripherals.

Describe the following implementations of the subsystem, considering the merit of each approach, in terms of design effort and cost effectiveness.

(a) a single chip using full custom design [6 marks]

(b) using semi-custom technologies [6 marks]

(c) using field programmable logic and standard parts [6 marks]

(d) using only standard parts [2 marks]