COMPUTER SCIENCE TRIPOS Part II (General)
DIPLOMA IN COMPUTER SCIENCE

Monday 30 May 1994 1.30 to 4.30

Paper 10 (Paper 1 of Diploma in Computer Science)

Answer **five** *questions.*
Submit the answers in five **separate** *bundles each with its own cover sheet.*
Write on **one** *side of the paper only.*

1 Modula-3

What is an `UNSAFE` module in Modula-3? [3 marks]

Compare and contrast the built-in Modula-3 procedures `VAL` and `LOOPHOLE`. [4 marks]

How is the following type `Bitset` likely to be implemented and why might this be useful?

```
TYPE
  Bitset = SET OF [0..31];
```
[3 marks]

A procedure `Swap` is required which takes a 6-digit Hexadecimal `CARDINAL` value as its single argument. The procedure is intended to treat the digits as three adjacent pairs and reverse the order of each pair. The output from the following statement should be `214365`:

```
Wr.PutText (Stdio.stdout, Fmt.Int(Swap(16_123456),16) & "\n");
```

Write a complete test program which incorporates this statement and a procedure `Swap` which exploits type `Bitset` and the procedure `LOOPHOLE`. Explain how the procedure works. [10 marks]

2 Modula-3

The following is a fragment of a Modula-3 program which is supplied with data arranged as one left-adjusted integer per line. The integers (which may be assumed to be all different) are read one at a time and the procedure `Put` assembles a simple binary tree by arranging for each integer in turn to be in a new node of the tree. The assembly process sorts the integers into numerical order.

```
VAR
  tree : PtrToNode := NIL;
BEGIN
  TRY
    LOOP
      TRY
        Put (Scan.Int (Rd.GetLine (Stdio.stdin)), tree)
      EXCEPT
        Scan.BadFormat =>
                       Wr.PutText (Stdio.stdout, "Bad datum\n")
      END
    END
  EXCEPT
    Rd.EndOfFile =>
  END;
  PrintTree (tree);
  Wr.Close (Stdio.stdout)
```

Explain the operation of the two `TRY`–`EXCEPT` clauses. [5 marks]

Provide a suitable `TYPE` statement to define the type `PtrToNode`. [5 marks]

Write a procedure `Put` and explain its operation. [5 marks]

The procedure `PrintTree` is intended to write out the values in the tree in ascending order. Write this procedure and explain its operation. [5 marks]

3 Further Modula-3

The thread system in Modula-3 uses *mutexes* and *condition variables* to control concurrency. An alternative scheme would be to provide *eventcounts* and *sequencers*. An eventcount is an integer, initially zero, equipped with the three atomic operations:

`advance` increments the count and returns its new value,

`read` returns the current value of the count, and

`await (value)` suspends the calling thread until the count is at least as large as the value given as an argument.

A sequencer is an integer, initially zero, equipped with a single atomic operation:

`ticket` increments the count and returns its previous value.

Given an eventcount, `guard`, and a sequencer, `turn`, a critical region can then be coded as follows:

```
myturn := turn.ticket ();
guard.await (myturn);
    .
    .    protected code
    .
EVAL guard.advance ();
```

Write an interface, `ECS`, defining opaque object types `EventCount` and `Sequencer`. `EventCount` should have methods `advance`, `read` and `await`, with appropriate signatures, and `Sequencer` should have a `ticket` method. [8 marks]

Sketch an implementation of the `ECS` module giving concrete revelations of the types and providing appropriate default methods. [12 marks]

4 Formal Languages and Automata

What is meant by the *language accepted* by a finite deterministic automaton $M = (Q, \Sigma, \delta, i, F)$? [2 marks]

Show that it is possible to associate with M a regular expression $\mathbf{r}$ over Σ denoting the same language as that accepted by M. [12 marks]

Illustrate your answer by constructing such a regular expression $\mathbf{r}$ when M is the finite deterministic automaton with

$$\begin{aligned} Q &= \{q_1, q_2, q_3\} \\ \Sigma &= \{0, 1\} \\ i &= q_1 \\ F &= \{q_3\} \end{aligned}$$

and with transition function δ defined by the table

	q_1	q_2	q_3
0	q_2	q_3	q_3
1	q_3	q_2	q_2

[6 marks]

5 Operating System Functions

Describe the use of an *inverted page table* for the implementation of virtual addressing and a paging virtual memory system. Give details of the operation of the page table and its associated data structures. [15 marks]

What can be done to overcome the thrashing problem encountered with a direct mapped inverted page table? [5 marks]

6 Operating Systems

An operating system supports multi-threaded processes. Within a given user-level address-space two threads cooperate by means of a shared, circular, N-slot buffer. Semaphores are supported by the language system.

Outline programs that may be executed by the thread which writes data into the buffer and the thread which reads data from it. [8 marks]

How would you ensure that several threads could write to, and read from, the buffer? [4 marks]

Explain how the semaphore implementation in the language system uses the thread implementation in the operating system. [8 marks]

7 Data Structures and Algorithms

For the following, n is a positive integer and G is a graph of N nodes (vertices) and E arcs (edges) each with a given weight (or cost). For *seven* of the following indicate, with a short justification, whether the statement is true or false.

(a) All functions f of the form $f(n) = An^k$ (with A and k being constants) are in the class $O(2^n)$.

(b) All sorting methods for an array of n elements take time $O(n^5)$.

(c) It is possible to sort an array of n elements using binary comparisons in $\Theta(n \log n)$ time.

(d) It is possible to sort an array of n elements using binary comparisons using $O(1)$ (i.e. constant independent of n) additional space.

(e) Radix sorting can sort any set of integers in linear time.

(f) All straight lines from the inside of a polygon to the outside intersect the points on the edges forming its boundary an odd number of times.

(g) It is always cheaper to find the shortest distance between two given nodes u, v of G than to find all N shortest distances from u to every other node.

(h) It is possible to find the shortest paths between all N^2 pairs of nodes of G in $O(N^3)$ time.

(i) If G is connected then the minimal spanning subtree of G contains the $N - 1$ edges whose weights are smallest.

(j) Given n points $(x_i, y_i), 1 \leqslant i \leqslant n$ in a plane, then the four points (x_a, y_a), (x_b, y_b), (x_c, y_c), (x_d, y_d) such that x_a is minimal of the x_i, x_b is maximal of the x_i, y_c is minimal of the y_i, y_d is maximal of the y_i form a quadrilateral Q which can be used to speed up a convex hull algorithm by preprocessing to remove points which lie inside Q.

Marks will be awarded for overall succinctness, attention to detail and absence of random guesses lacking justification.

[20 marks]

8 Data Structures and Algorithms

Compare and contrast three implementations for a priority queue in terms of an explanation of: what the data structures represent; a sketch of the principal routines; and $O(f(n))$ timings when it is implemented as

(a) an (unsorted) array [5 marks]

(b) a sorted array [5 marks]

(c) a heap [10 marks]

You should consider the routines *insert*, *extract* and *test-for-emptiness* for a priority queue holding n elements. Consider also a routine to initialise a priority queue to hold a given set of n elements.

9 Graphics

Explain with a diagram how a shadow mask cathode ray tube works. [12 marks]

What might be the point of extending the scheme to accommodate five electron guns? [8 marks]

10 Numerical Analysis I

The mid-point rule can be expressed in the form

$$I_n = \int_{n-\frac{1}{2}}^{n+\frac{1}{2}} f(x)\,dx = f(n) + e_n$$

where

$$e_n = f''(\theta_n)/24$$

for some θ_n in the interval $(n - \frac{1}{2}, n + \frac{1}{2})$. Assuming that a formula for $\int f(x)\,dx$ is known, and using the notation

$$S_{p,q} = \sum_{n=p}^{q} f(n),$$

describe a method for estimating the sum of a slowly convergent series $S_{1,\infty}$, by summing only the first N terms and estimating the remainder by integration. [7 marks]

Assuming that $f''(x)$ is a positive decreasing function, derive an estimate of the error $|E_N|$ in the method. [5 marks]

Given

$$\int \frac{dx}{1+x^2} = \tan^{-1} x,$$

apply the method to

$$\sum_{n=1}^{\infty} \frac{1}{1+n^2}.$$

What is the integral remainder to be added to $S_{1,N}$? [4 marks]

To the nearest power of 10, how large should N be to achieve an absolute error of approximately 10^{-16}? [4 marks]

11 Discrete Mathematics

Consider finite strings over the alphabet $\{A, B, C\}$. Say that a string is *valid* if it does not contain either of the substrings AA, AB.

List the *invalid* strings of length 3. [4 marks]

Let $\upsilon(n)$ be the number of valid strings of length n. Show that for all $n \geqslant 0$

$$\upsilon(n+2) = 2.\upsilon(n+1) + \upsilon(n).$$ [7 marks]

Hence determine a general formula for $\upsilon(n)$. [9 marks]

12 Introduction to Functional Programming

Recall that `f o g` is the function that maps `x` to `f(g(x))`. Consider the ML definitions

```
fun I x = x;
fun pair (f,g) (x,y) = (f x, g y);
fun pup (f,g) z = (f z, g z);
fun fst (x,y) = x;
fun snd (x,y) = y;
```

Describe the effect of the following functions:

```
pair(I,I)                pair(f1 o f2, g1 o g2)
pup(fst,snd)             pup(f o fst, g o snd)
```
[4 marks]

Infinite lists can be represented in a functional language by triples. A triple of the form (a, h, t) represents the infinite list whose nth element is $h(t^n(a))$ for $n \geqslant 0$.

(a) Give a representation for the infinite list $n, n+1, n+2, \ldots$ [2 marks]

(b) Code in ML a map functional for this representation; given a function f and the infinite list $x_0, x_1, \ldots$, it should yield the representation of $f(x_0), f(x_1), \ldots$ [3 marks]

(c) Code in ML a zip function, which combines the infinite lists $x_0, x_1, \ldots$ and $y_0, y_1, \ldots$ to the list of pairs $(x_0, y_0), (x_1, y_1), \ldots$ [4 marks]

(d) Code in ML an interleave function, which combines the infinite lists $x_0, x_1, \ldots$ and $y_0, y_1, \ldots$ to yield $x_0, y_0, x_1, y_1, \ldots$ [5 marks]

(e) How does this representation compare with the usual representation of infinite lists in ML? Briefly discuss. [2 marks]