1993 Paper 8 Question 10

Semantics

State the principle of fixed point induction. [4 marks]

D is a cpo of 'integer streams': it comes equipped with a continuous function $in : (\mathbb{Z} \times D)_\perp \to D$ that possesses a continuous inverse $out : D \to (\mathbb{Z} \times D)_\perp$. (Thus the composition of in and out in either order is the appropriate identity function.) Moreover D has the property that the identity function $id_D \in (D \to D)$ is the least fixed point of the continuous function $\delta : (D \to D) \to (D \to D)$ which maps $f \in (D \to D)$ to $\delta(f) \in (D \to D)$, where for each $d \in D$

$$\delta(f)(d) = \begin{cases} in([(n, f(x))]) & \text{if } out(d) = [(n, x)] \\ in(\perp) & \text{if } out(d) = \perp \end{cases}$$

Let $mapS : D \to D$ be a continuous function satisfying that for all $d \in D$

$$mapS(d) = \begin{cases} in([(n+1, mapS(x))]) & \text{if } out(d) = [(n, x)] \\ in(\perp) & \text{if } out(d) = \perp \end{cases}$$

Using fixed point induction for δ, show that there is at most one solution $d \in D$ to the equation

$$d = in([(0, mapS(d))])$$

Hint: if d_1 and d_2 are both solutions, consider the property of $f \in (D \to D)$ given by '$f\ mapS = mapS\ f$ and $f(d_1) \sqsubseteq d_2$'. [16 marks]