1993 Paper 5 Question 9

Foundations of Functional Programming

Describe how the λ-calculus models the operations of addition, test for zero and successor, representing the natural numbers by Church numerals. [4 marks]

The Fibonacci sequence is defined by $F_0 = 0$, $F_1 = 1$ and $F_k = F_{k-1} + F_{k-2}$ for $k \geqslant 2$. Present a λ-term **fib** that computes the Church numeral for F_k given the Church numeral for k, for all $k \geqslant 0$. Do not use **Y** or any other fixed point combinator. You may take as primitive the λ-calculus encodings of standard data structures. [6 marks]

Describe how to assign Gödel numbers to λ-terms and explain the notation $\ulcorner M \urcorner$. Describe an application of these techniques. [3 marks]

Present a λ-term **iszero**, such that

$$\mathbf{iszero}\ulcorner M \urcorner = \begin{cases} \mathbf{true} & \text{if } M = \underline{0} \\ \mathbf{false} & \text{if } M \neq \underline{0} \end{cases}$$

or prove that no such term exists. [7 marks]