1993 Paper 4 Question 8

Data Structures and Algorithms

A directed graph of n nodes numbered $1, 2, \ldots, n$ can be represented by an $n \times n$ adjacency matrix G_1, where $G_1[i, j]$ is true if there is an edge connecting node i to node j, and $G_1[i, j]$ is false otherwise.

By extension, define G_k to be that matrix such that $G_k[i, j]$ is true if there is a path of length $\leqslant k$ connecting node i to node j, and $G_k[i, j]$ is false otherwise.

Describe an algorithm to generate G_2 from G_1. [12 marks]

How could this algorithm be used to generate the transitive closure of a graph given its adjacency matrix? [5 marks]

What is the cost of this transitive closure algorithm in terms of n and m, where m is the maximum path length in the transitive closure? [3 marks]