

1993 Paper 1 Question 8

The structure of a binary tree with integers at the leaves is represented by the following ML datatype:

```
datatype T = n of int | d of T*T;
```

Define a function `flatten` which when given an argument `t` of type `T` will yield the list of integers obtained by a left to right walk over `t`. For example,

```
flatten (d(d(n1,n2),n3)) = [1,2,3].
```

Define a function `splits` which, when given a list of length $n > 0$, will yield the list of 2-tuples representing the $n - 1$ ways of splitting the given list into two non-empty sublists. For example,

```
splits [1,2,3] = [ ([1], [2,3]), ([1,2], [3]) ].
```

Hence or otherwise define a function `alltrees` which, when given a list of length $n > 0$, will form a list of all the trees of type `T` that will flatten to the given list. For example,

```
alltrees [1,2,3] = [ d(n1, d(n2, n3)), d(d(n1, n2), n3) ].
```