1993 Paper 11 Question 11

Discrete Mathematics

Let A be a non-empty set, and $\prec$ be a relation on A. What is meant by saying that $(A, \prec)$ is a partially ordered set? [3 marks]

What additional conditions must be satisfied if $(A, \prec)$ is to form:

(a) a totally ordered set [1 mark]

(b) a well-ordered set [2 marks]

(c) a complete partially ordered set? [3 marks]

Suppose now that A is a non-empty set, R a relation on A, and $B \subseteq A$ a non-empty subset. Write $R_B = R \cap (B \times B)$ for the relation induced on B by R. Show that if $(A, \prec)$ is a partially ordered set, so also is $(B, \prec_B)$. [1 mark]

On the set $\mathbb{Z} = \{0, \pm 1, \pm 2, \ldots\}$ of integers define the following relations:

(i) $\leqslant = S^*$, the reflexive transitive closure of $S = \{(n, n+1) : n \in \mathbb{Z}\}$

(ii) $d = \{(m, n) : \exists\, q \in \mathbb{Z} \text{ such that } mq = n\}$

For each of the set $\mathbb{Z}$ and its subsets $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$, $\mathbb{N}^+ = \{1, 2, 3, \ldots\}$ say whether the relations $\leqslant$ and d induce a partial ordering. Identify instances in which any of the cases (a)–(c) arises, giving your reasons briefly. [10 marks]