Let $g : A \to B$ be a function with domain $A$ and range $B$. Show that the relation $R$ defined by

$$xRy \iff g(x) = g(y)$$

is an equivalence relation on $A$. [4 marks]

Let $f(n, r)$ be the number of surjections from a set $A$ having $n$ elements to a set $B$ having $r$ elements. Show that

$$f(n, r) = r(f(n - 1, r - 1) + f(n - 1, r)).$$

[8 marks]

Evaluate $f(n, r)$ in the cases:

(a) $r = 2$ [3 marks]

(b) $r = (n - 1)$ [5 marks]