COMPUTER SCIENCE TRIPOS Part II

Tuesday 1 June 1993 1.30 to 4.30

Paper 7

Answer **five** *questions.*
Submit the answers in five **separate** *bundles each with its own cover sheet.*
Write on **one** *side of the paper only.*

1 Distributed Systems

A distributed software system follows the client-server model. The microkernel on which it is based supports multi-threaded processes. A remote procedure call (RPC) package is used for client-server interactions. The RPC system runs above an unreliable, datagram-based communications service.

(*a*) Explain how timers may be used in the RPC protocol to achieve client-server synchronisation. [10 marks]

(*b*) Discuss how the RPC system may support the location of remote procedures. [7 marks]

(*c*) Discuss the requirements on the RPC system that follow from the use of multi-threaded processes. [3 marks]

2 Distributed Systems

A distributed computation may involve related operations on a number of objects which reside at different nodes of a distributed system.

(*a*) Explain why the concept of *transaction* is suitable for modelling such a computation. [6 marks]

(*b*) Explain what is involved in committing a transaction in a distributed system. [14 marks]

3 Comparative Architectures

A revisionist view might be that a processor design is RISC if every transistor pays its way in terms of global system performance, i.e. the design is in some sense a local optimum. Explain how this view relates to the original definition of RISC. Can a CISC instruction set be RISC under such a definition? [4 marks]

Given a certain silicon budget (e.g. 10^6 transistors) compare and contrast, under the above criterion, alternative ways of spending the excess over the (say) 10^4 transistor cost of a simple load-store one-accumulator machine. You might find it helpful to consider the various features and instructions of common processors. [12 marks]

This budget is sufficient to build most, but not all, of a VAX. Which parts would you omit and how could you arrange to execute full VAX code? [4 marks]

4 Comparative Architectures

Compare and contrast possible implementations of instructions which

(*a*) load a single byte from memory [8 marks]

(*b*) store a single byte in memory [4 marks]

(*c*) move a sequence of n bytes from one address to another [8 marks]

For each implementation, indicate briefly whether it conforms to the RISC philosophy, its effect on pipelining, and its likely efficiency compared with any alternatives.

5 Specification and Verification of Hardware

You are given components MUX, REG and COMP whose behaviour is defined by

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MUX(sw,in1,in2,out) = (∀ t. out t = (sw t → in1 t | in2 t))

REG v (in,out) = (out 0 = v) ∧ (∀ t. out(t+1) = in t)

COMP(in1,in2,out) = (∀ t. out t = (in1 t < in2 t))
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Use these to implement a device MAX that satisfies the specification

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MAX(in,out) ⇒ (∀ t. out t = Max in t)
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where the function Max is defined by

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(Max in 0 = in 0) ∧
(Max in (n+1) = (Max in n < in(n+1) → in(n+1) | Max in n))
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[10 marks]

Prove that your implementation meets its specification. [10 marks]

6 Specification and Verification of Hardware

Describe the simple switch model of CMOS transistors. [5 marks]

Draw a circuit diagram of a CMOS inverter and give a proof that it is correct in the simple switch model. [2 + 3 marks]

Describe the unidirectional model of NMOS transistors. [5 marks]

Draw a circuit diagram of an NMOS inverter and give a proof that it is correct in the unidirectional model. [2 + 3 marks]

7 Numerical Analysis II

State a recurrence formula suitable for evaluating the sequence of Chebyshev polynomials $\{T_n(x)\}$ for an argument x. What are the starting values? [2 marks]

The error in Lagrange interpolation can be expressed in the form

$$f(x) - L_{n-1}(x) = \frac{f^{(n)}(\zeta)}{n!} \prod_{j=1}^{n} (x - x_j)$$

for a suitable function $f(x)$. Suggest a choice of the interpolation points $\{x_j\}$ which tends to minimise this error over the interval $[-1, 1]$. [3 marks]

Hence justify and explain the method of *economisation of a power series*. [5 marks]

In what sense is an economised power series a *best approximation*? [2 marks]

Suppose $P_n(x)$ is a polynomial formed by truncating a power series after the term in x^n. Perform an economisation of the truncated power series

$$\cosh x \simeq P_4(x) = 1 + \tfrac{x^2}{2!} + \tfrac{x^4}{4!}$$ [5 marks]

Given that the maximum error in $P_4(x)$ over $[-1, 1]$ is approximately 0.0014, compare the error in your economised polynomial with the error in $P_2(x)$. [3 marks]

8 Numerical Analysis II

If $\mathbf{B}$ is a real symmetric $n \times n$ matrix such that $\bar{\mathbf{z}}^T \mathbf{B} \mathbf{z} \geqslant 0$ for any complex vector $\mathbf{z}$, prove that any eigenvalue λ of $\mathbf{B}$ is such that $\lambda \geqslant 0$. Hence prove that the eigenvalues of $\mathbf{A}^T \mathbf{A}$, where $\mathbf{A}$ is any real square matrix, are real and non-negative. [3 marks]

Let $\mathbf{P}$, $\mathbf{Q}$ be real $n \times n$ matrices and let $\|\mathbf{P}\|_2^2$ denote the maximum eigenvalue of $\mathbf{P}^T \mathbf{P}$. State Schwarz's inequality for $\|\mathbf{PQ}\|_2$. Explain how this is modified if $\mathbf{Q}$ is replaced by a vector of n elements. [3 marks]

Derive the *condition number* K for solution of the equations $\mathbf{Ax} = \mathbf{b}$. Hint: start by setting $\mathbf{e} = \mathbf{x} - \hat{\mathbf{x}}$ where $\hat{\mathbf{x}}$ is an approximate solution. [5 marks]

Describe the *singular value decomposition*

$$\mathbf{A} = \mathbf{UWV}^T$$

and explain how you would use it to solve the n equations $\mathbf{Ax} = \mathbf{b}$ when $\mathbf{W}$ has rank n. [5 marks]

How may the singular value decomposition help in solving the equations $\mathbf{Ax} = \mathbf{b}$ when $\mathbf{A}$ has rank $< n$? Use the case $n = 4$, $\mathbf{W} = \mathrm{diag}\{1, 10^{-3}, 10^{-20}, 0\}$ to illustrate your answer. (You may assume that *machine epsilon* $\simeq 10^{-16}$.) [4 marks]

9 Graphics II

When scan-converting items for display, a Z-buffer is sometimes used to avoid some sorting. Outline its operation and limitations. [12 marks]

The use of an A-buffer will improve matters. Explain why. [8 marks]

10 Semantics

An imperative language has boolean expressions be, integer expressions ie, and commands C, whose abstract syntax is specified by:

$$\begin{aligned} ie &::= \underline{n} \mid X \mid ie + ie \mid ie - ie \\ be &::= \underline{b} \mid ie = ie \\ C &::= \mathsf{skip} \mid X := ie \mid C \,;\, C \mid \mathsf{if}\ be\ \mathsf{then}\ C\ \mathsf{else}\ C \mid \mathsf{while}\ be\ \mathsf{do}\ C \end{aligned}$$

where b is *true* or *false*, n is any integer, and X ranges over a fixed set of variables. Describe the operational semantics of the language in terms of inductively defined evaluation relations

$$be, S \Rightarrow b \qquad ie, S \Rightarrow n \quad \text{and} \quad C, S \Rightarrow S'$$

where S and S' are integer-valued functions on the set of variables. [5 marks]

In what sense are these evaluation relations *deterministic*? What is meant by the assertion that two commands are *semantically equivalent*? [3 marks]

For any choice of be, C and C', which of the following pairs of commands are semantically equivalent and which are not? Justify your answer in each case.

(a) $((\mathsf{while}\ be\ \mathsf{do}\ C) \,;\, C)$ and $(\mathsf{if}\ be\ \mathsf{then}\ ((\mathsf{while}\ be\ \mathsf{do}\ C) \,;\, C)\ \mathsf{else}\ C)$

(b) $(C \,;\, (\mathsf{while}\ be\ \mathsf{do}\ C))$ and $(\mathsf{if}\ be\ \mathsf{then}\ (\mathsf{while}\ be\ \mathsf{do}\ C)\ \mathsf{else}\ C)$

(c) $(\mathsf{while}\ be\ \mathsf{do}\ (\mathsf{if}\ be\ \mathsf{then}\ C\ \mathsf{else}\ C'))$ and $(\mathsf{while}\ be\ \mathsf{do}\ C)$

[12 marks]

11 Additional Topics

Describe, for a competent programmer, the decoding of a Lempel-Ziv data compression code. Assume that the input is a stream of positive integers. [10 marks]

Construct a Huffman Code for a seven-letter alphabet, given the following approximate frequencies:

a	b	c	d	e	f	g
192	58	105	71	315	131	0

[4 marks]

Give an error-correcting code and explain how it works, excluding the simple repetition code. [3+3 marks]

12 Additional Topics

Explain the distinction between Public- and Secret-Key cryptosystems. [4 marks]

Name an example of each type. [1+1 marks]

Is a One Time Pad a Public- or Secret-Key system? [1 mark]

Discuss the advantages and disadvantages of One Time Pads as compared with alternative systems. When might One Time Pads be used and when should they be avoided? [13 marks]