COMPUTER SCIENCE TRIPOS Part IB

Monday 31 May 1993 1.30 to 4.30

Paper 3

Answer **five** *questions.*
Submit the answers in five **separate** *bundles each with its own cover sheet.*
Write on **one** *side of the paper only.*

1 Further Modula-3

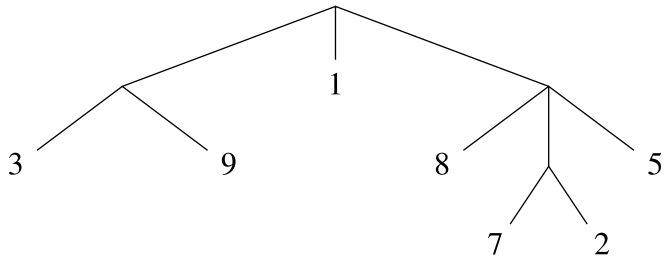
Describe the `MUTEX` type and the `LOCK..DO..END` construction in Modula-3 for restricting concurrent access to critical regions. Illustrate your answer with an example. [6 marks]

Show how the `LOCK..DO..END` construction is expanded by the compiler into calls to the `Thread` library and some exception handling. [4 marks]

The built-in facilities for restricting concurrency in Modula-3 allow only one thread at a time to be executing within the critical region. A different approach is to distinguish *shared* and *exclusive* access to a critical region; any number of *readers* may share access at the same time but only one *writer* may acquire exclusive access, also excluding any readers while it does so. Define a `MultiMutex` type and sketch suitable library procedures to implement such a scheme. [10 marks]

2 Common Lisp

Consider trees that have two kinds of nodes. A node is either a **leaf**, labelled by a number, or a **branch**, and has one or more subtrees. For example:



One imagines that the edges from each branch node are numbered from left to right starting from 0. A list of these numbers thus designates the path from the root to a node. In the tree shown above, the path (2 1 1) designates the path to the node labelled 2.

(*a*) Describe a good representation for such trees in Lisp. [3 marks]

(*b*) Write a Lisp function `getnode` such that `(getnode` *path tree*`)` returns the node of *tree* designated by *path*, assuming that the tree contains such a node. [5 marks]

(*c*) Write a Lisp function `maxpath` such that `(maxpath` *tree*`)` returns the maximum of the leaf nodes in the tree, together with the path to that node. For the tree shown above, `maxpath` should return 9 as the maximum and (0 1) as the path. [12 marks]

3 Foundations of Logic Programming

Consider the following set of clauses, where x is a variable and b is a Skolem constant:

$$\{\neg P(x), \neg Q(x), P(f(x))\} \tag{1}$$

$$\{\neg P(x), Q(x)\} \tag{2}$$

$$\{P(b)\} \tag{3}$$

$$\{\neg P(f(f(f(f(x)))))\} \tag{4}$$

(*a*) How many resolution steps are required to derive the empty clause from these clauses? Justify your answer clearly. [5 marks]

(*b*) Prolog uses resolution in a restricted form. How many Prolog-style resolution steps are required to derive the empty clause, taking (4) as the goal clause and (1)–(3) as program clauses? Justify your answer clearly. [5 marks]

(*c*) How do a resolution theorem-prover and a Prolog system differ in their implementation of resolution? [4 marks]

(*d*) The theorem $(\forall y)(\exists x)\neg(p(x,y) \leftrightarrow \neg((\exists z)(p(x,z) \wedge p(z,x))))$ is to be proved using the resolution method. Convert the problem to clause form, showing all steps. (Omit the resolution proof itself.) [6 marks]

4 Prolog

The following Prolog clauses define the procedures named `perm` and `select`. The goal `perm(X,Y)` succeeds for the list `X`, instantiating `Y` to a permutation of list `X`. Successive backtrackings will enumerate each possible permutation. For example, evaluating the goal `perm([a,b,c],Q)` will instantiate `Q` successively on each backtracking to: `[a,b,c]`; `[a,c,b]`; `[b,a,c]`; `[b,c,a]`; `[c,a,b]`; `[c,b,a]`.

```
select(H,[H|T],T).
select(H,[N|T],[N|L]) :- select(H,T,L).

perm(X,[H|T]) :- select(H,X,Z), perm(Z,T).
perm([],[]).
```

Explain how procedures `perm` and `select` work, using a small example. [20 marks]

5 Programming Language Compilation

Discuss the merits of translating the abstract syntax tree representation of a program into assembly language by means of

(*a*) an *ad hoc* recursive tree walking program [10 marks]

(*b*) an algorithm based on tables automatically generated from tree translation rules [10 marks]

6 Operating System Functions

Describe the functionality you would expect to find in the file system directory service of a multi-user operating system. [10 marks]

Describe two ways in which multiple names for the same file can be supported, and what problems arise as a result. [10 marks]

7 Data Structures and Algorithms

A *strictly binary tree* is a binary tree in which every node that is not a leaf has two children. Suppose that for a strictly binary tree there exists $c > 1$ such that the ratio of the lengths of any two root-to-leaf paths is no greater than c.

For a tree of height h, derive the upper and lower bounds on N, the number of nodes in the tree. [15 marks]

Suppose instead that every node that is not a leaf has n children. What then would be the upper and lower bounds? [5 marks]

8 Data Structures and Algorithms

For a given problem with inputs of size n, algorithms A, B, C are executed. In terms of running time, one of the algorithms is $O(n)$, one $O(n \log n)$ and one $O(n^2)$. Some measured running times of these algorithms are given below:

		INPUT SIZE		
		512	1024	2048
ALGORITHM	A	70	134	262
	B	135	517	2053
	C	42	86	182

Identify which algorithm is which and explain the observed running times. Which algorithm would you select for different values of n? [20 marks]

9 Graphics I

RasterOp is the name given to an operation which generates, from a number of rasters of pixel values, another raster of pixel values.

Describe suitable versions that could be used

(a) to move a window on screen while preserving background [12 marks]

(b) to blend two images in proportions given by a mask [8 marks]

10 Numerical Analysis I

In the IEEE binary standard (*IEEE* 754) what do the parameters p (precision), e_{min} and e_{max} specify? How is the value of an exponent e stored? [3 marks]

Explain the terms *normalised number*, *denormal number*, *hidden bit* and *NaN*. [4 marks]

In terms of the stored bit-pattern, how can each of the following be recognised: ± 0, $\pm\infty$, denormal number, NaN. [4 marks]

Suppose for some special-purpose hardware that a floating-point implementation is to be provided using only one byte for each representable number. Suppose also that, as far as possible, the principles of IEEE binary arithmetic are to be adhered to. If a sign bit of 0 represents a positive number, $p = 4$, $e_{min} = -2$ and $e_{max} = 3$ what should the following bit patterns represent?

00000000   00000001   00110000   11110000   11110001 [5 marks]

Consider the evaluation under IEEE arithmetic of the functions

$$(a) \quad \frac{x^2+2}{x^2+1} \qquad (b) \quad \ln(x^2+2)$$

where x and the function values are representable numbers, but x^2 is not. Show how you would formulate the evaluation of (a) and (b) to avoid this problem. [4 marks]