Optimising Compilers Exercise Sheet 3

The purpose of this exercise is to gain familiarity with *constraint-based analyses*, particularly *0CFA* (zeroth-order control-flow analysis).

Questions

1. (a) What is a higher-order function?
   (b) How do higher-order functions make it harder to predict control flow within a program?
   (c) How does the 0CFA help to predict control flow?
   (d) Do object-oriented programs have analysis issues related to higher-order functions?

Consider the following simple λ-calculus like language, call it $\mathcal{L}$:

$$e ::= v \mid c \mid \lambda v.e \mid e_1 e_2 \mid \texttt{let}\ \ v = e_1\ \ \texttt{in}\ \ e_2 \mid \texttt{if}\ \ e_1\ \ \texttt{then}\ \ e_2\ \ \texttt{else}\ \ e_3 \mid e_1 \oplus e_2$$

where v ranges over variables, c ranges over integer constants, and $\oplus$ ranges over binary operations.

0CFA computes information about control flow in a program by computing a subset of a program's data flow: the flow of functions (or function pointers). In the following, the data flow of integer constants will also be tracked to aid understanding.

2. (a) Define informally the notion of a *binding site* and *use site* and indicate the binding and use sites in the syntax of $\mathcal{L}$.
   (b) The following expression has a *single program point* labelling the formal parameter x of f:
   $$\texttt{let}\ \ f = \big(\lambda x^0.x + x\big)\ \ \texttt{in}\ \ f\ 2 + f\ 3$$
   Label the remaining program points (it may help to write the expression as a tree).
   (c) Given *flow variables* α_i associating sets to each program point, what is the value of set α_0 following a 0CFA? What integer values flow out of the body of the λ?
   (d) Write down and explain the rule for generating constraints for **let**-bindings and variables v.
   (e) Consider the following expression with a partial labelling of program points:
   $$\texttt{let}\ \ f = \big(\lambda x.x^1\ 0\big)\ \ \texttt{in}\ \ \big(\texttt{let}\ \ g = \big(\lambda y^0.y + 1\big)\ \ \texttt{in}\ \ (f\ g) + (g\ 1)\big)$$
   Compute the flow sets for α_1 and α_0.

3. (a) Calculate a full 0CFA (tracking just function values, not integer values) for the following expression:
   $$\texttt{let}\ \ f = (\lambda x.x\ 0)\ \ \texttt{in}\ \ \big(f\,(\lambda y.y * 3)\big) + \big(f\,(\lambda z.z + 1)\big)$$
   (b) Write down and explain the rule for generating constraints for functions and function application.

Suggested past exam questions

2004 Paper 9 Question 3

2007 Paper 9 Question 16

Relevant past exam questions

This section contains links to all past exam questions relevant to the topics covered in this supervision sheet. Note that some questions appear under multiple headings and / or on multiple exercise sheets when they cover more than one topic.

- 2015 Paper 9 Question 9
- 2007 Paper 9 Question 16
- 2004 Paper 9 Question 3
- 1998 Paper 9 Question 7