

DSP COMPUTER SCIENCE TRIPOS Part II

Monday 27 October 2025 12:00 to Monday 3 November 2025 12:00

Module DSP – Digital Signal Processing – Assignment 1

This assignment involves programming. The recommended programming language is Julia and library functions referred to in the problems may be found in the Julia packages `DSP.jl`, `FFTW.jl` and `Plots.jl`. [Implementations in other suitable languages, using equivalent library functions, such as MATLAB's Signal Processing Toolbox, or the Python packages `matplotlib` and `scipy.signal`, are also acceptable.]

Prepare the solutions and answers to all parts as a single PDF file and include all source code written, along with any required outputs produced by the programs. A `Pluto.jl` notebook provides a convenient way to combine answer text, Julia source code and outputs into a single PDF. Proofread the PDF, not just the notebook, and try to keep it brief.

Submit your work via

<https://www.vle.cam.ac.uk/course/view.php?id=256758>

no later than 12:00 on Monday 3 November 2025.

Students may be required to sign an undertaking that work submitted will be entirely their own; no collaboration is permitted.

1 (a) Julia commands (similar to)

```
using DSP, Plots
plot_filter(x, y; kwargs...) =
    plot([x y], linecolor=[:blue :red], markercolor=[:blue :red],
         markershape=[:x :circle], markerstrokewidth=3,
         size = (800, 250), xticks=[], yticks=[0],
         label=["x" "y"], ylim=[-7,7]; kwargs...);
x = [ 0,  0,  0, -4,  0, 0,  0,  0,  0, 0, 2,  2, 2, 2,
      2,  0, -3, -3, -3, 0,  0,  0,  0, 0, 1, -4, 0, 4,
      3, -1,  2, -3, -1, 0,  2, -4, -2, 1, 0,  0, 0, 3,
      -3,  3, -3,  3, -3, 3, -3,  3, -3, 0, 0,  0, 0, 0 ];
y = filt([1, 1, 1, 1]/4, [1], x);
plot_filter(x, y; title="4-point moving average system")
```

produced the plot on slide 19 to illustrate the 4-point moving average system. The DSP.jl library function `filt(b, a, x)` applies to the finite sequence x the discrete system defined by the constant-coefficient difference equation with coefficient vectors b and a (see slide 25).

Change in this program the `filt` parameters to implement instead the

(i) exponential averaging system (slide 20)

(ii) accumulator system (slide 21)

(iii) backward difference system (slide 22)

and provide the coefficient vectors b and a for each of these systems.

[*Note:* A function equivalent to `filt` is in MATLAB called `filter` and in Python `scipy.signal.lfilter`.]

(b) (i) Simulate the reconstruction a sampled frequency-limited signal, following these steps:

- Generate a one second long Gaussian noise sequence r with a sampling rate of 300 Hz, where each sample is independent and identically distributed and drawn from a normal distribution, using the `randn` function.
- Taper the noise sequence r by setting its first and last 15 samples to zero.
- Use the `DSP.jl` function call

```
digitalfilter(Lowpass(45),  
              FIRWindow(Windows.hamming(51)); fs=300)
```

to design a finite impulse response low-pass filter with a cut-off frequency of 45 Hz. This function will return a vector b for use in a digital filter of the type shown on slide 25. What vector a is required in addition?

[*Note:* In MATLAB an equivalent function call is `fir1(50, 45/150)` and in Python `scipy.signal.firwin(51, 45/150)`.]

Use the `filtfilt` function in order to apply that filter to the generated noise signal, resulting in the filtered noise signal x . (This function applies the filter twice, once in forward and once in backward direction.)

- Then sample x at 100 Hz by setting all but every third sample value to zero, resulting in the (equally long) sequence y .
- Implement sinc interpolation with a suitably scaled sinc function (and any required loops) to reconstruct the zeroed samples of y , resulting in a reconstructed sequence z .
- Generate another low-pass filter with `digitalfilter`, with a cut-off frequency of 50 Hz and apply it with `filtfilt` to y , resulting in interpolated sequence u . Multiply the result by three, to compensate the energy lost during sampling.
- Plot x , y , z and u , all on top of each other (superimposed) in one figure, and compare x with z and u .
- Lastly, turn the calls to `digitalfilter` and `Windows.hamming` into comments and replace them with your own implementations.

(ii) Why should the first filter have a lower cut-off frequency than the second?

(c) (i) Simulate the reconstruction of a sampled band-pass signal, with these steps:

- Generate a 1 s noise sequence r , as in part (b)(i), but this time use a sampling frequency of 3 kHz. Set the first and last 500 samples to zero.
- Apply to that with `filtfilt` a band-pass filter that attenuates frequencies outside the interval 31–44 Hz, resulting in filtered sequence x . The following DSP.jl function call will design such a filter for you:

```
digitalfilter(Bandpass(31, 44),
              Chebyshev2(3, 30); fs=3000)
```

This function returns a filter object that can be passed directly to `filt` or `filtfilt`. [*Hint:* To obtain a representation as coefficient vectors b and a , you can convert that filter object to type `PolynomialRatio` and then use functions `coefa` and `coefb` on that.]

[*Note:* In MATLAB an equivalent function call is `cheby2(3, 30, [31 44]/1500)`, which returns *two* vectors b and a . In Python: `scipy.signal.cheby2(3,30, [31/1500,44/1500], "bandpass")`.]

- Then sample the resulting signal at 30 Hz by setting all but every 100-th sample value to zero, resulting in y .
- Implement sinc interpolation with a suitably scaled *and modulated* sinc function (and any required loops) to reconstruct an approximation of x from y , resulting in a reconstructed sequence z .
- Generate with

```
digitalfilter(Bandpass(30, 45),
              Chebyshev2(3, 20); fs=3000)
```

another band-pass filter for the interval 30–45 Hz and apply it to y , to reconstruct the original signal as sequence u . (What factor do you have to multiply it by, to compensate the energy lost during sampling?)

- Plot x , y , z , and u , all on top of each other in one figure, and compare the original band-pass signal x and the two reconstructed versions z and u after being sampled at 30 Hz.

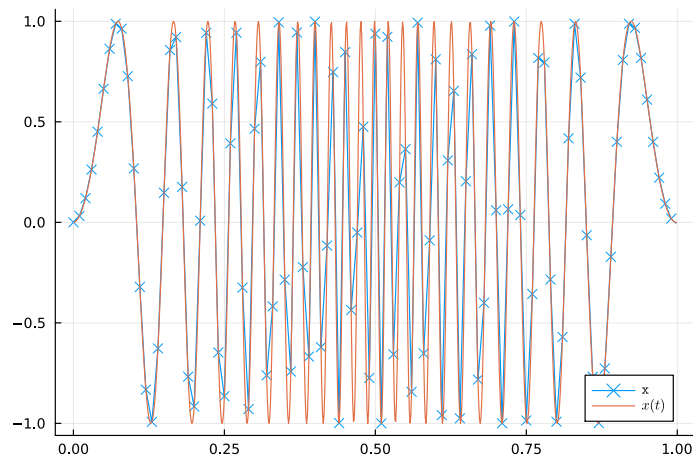
(ii) Why does the reconstructed waveform differ much more from the original if you reduce all the cut-off frequencies of all band-pass filters by 5 Hz?

- (d) Write a function `fft_interp(x, f)` that receives a vector `x` of real-valued floating-point numbers and an integer `f`. You can assume that the vector `x` contains the result of sampling a continuous function $x(t)$ that contains only frequencies less than half the sampling frequency used to obtain `x`. The function should return a vector that is `f` times longer than `x` and represents approximately the same continuous signal $x(t)$ as `x`, but sampled with a factor `f` higher sampling frequency.

In your function, use the Fast Fourier Transform to efficiently approximate sinc interpolation. After applying any necessary padding to `x`, use the `fft` function to convert that finite discrete sequence into a frequency-domain representation `X`. Then modify `X` such that it resembles the frequency-domain representation of the same signal $x(t)$ if it had been sampled at an `f`-times higher sampling frequency. Finally apply the `ifft` function to return to the time domain, remove any padding and return the result.

Generate two test signals `x` to demonstrate this signal, each 100 samples long:

- (i) a unit impulse at the 10th sample, all other samples being zero;
- (ii) a sine-wave of variable frequency that starts with value 0 and with a normalized frequency of 0 rad/sample, then increases during the first half of the signal (over 50 samples) in frequency linearly until it reaches a normalized frequency of 0.9π rad/sample, and then decreases again linearly down to 0 rad/sample during the second half of the signal.



For each of these test signals, prepare a plot that shows both `x` and `fft_interp(x, 8)` plotted on top of each other (superimposed) as lines, such that the corresponding samples are aligned with each other both horizontally and vertically.

END OF PAPER