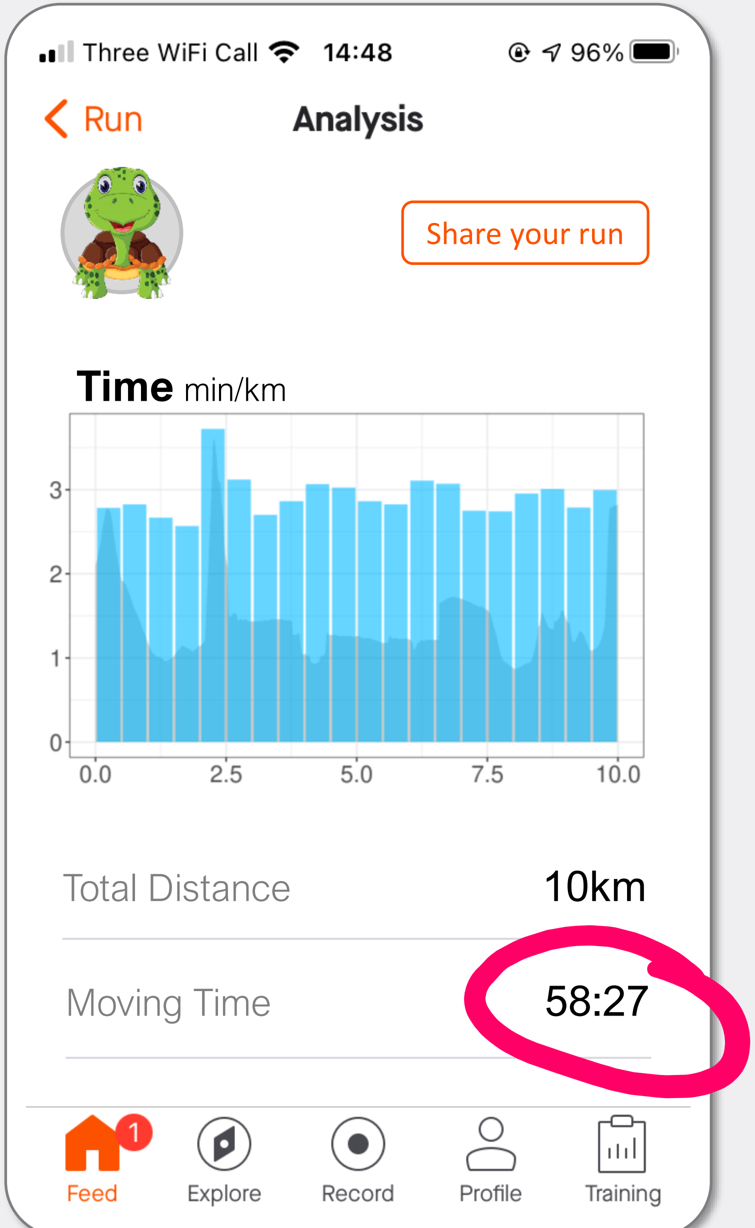
SECTION 7

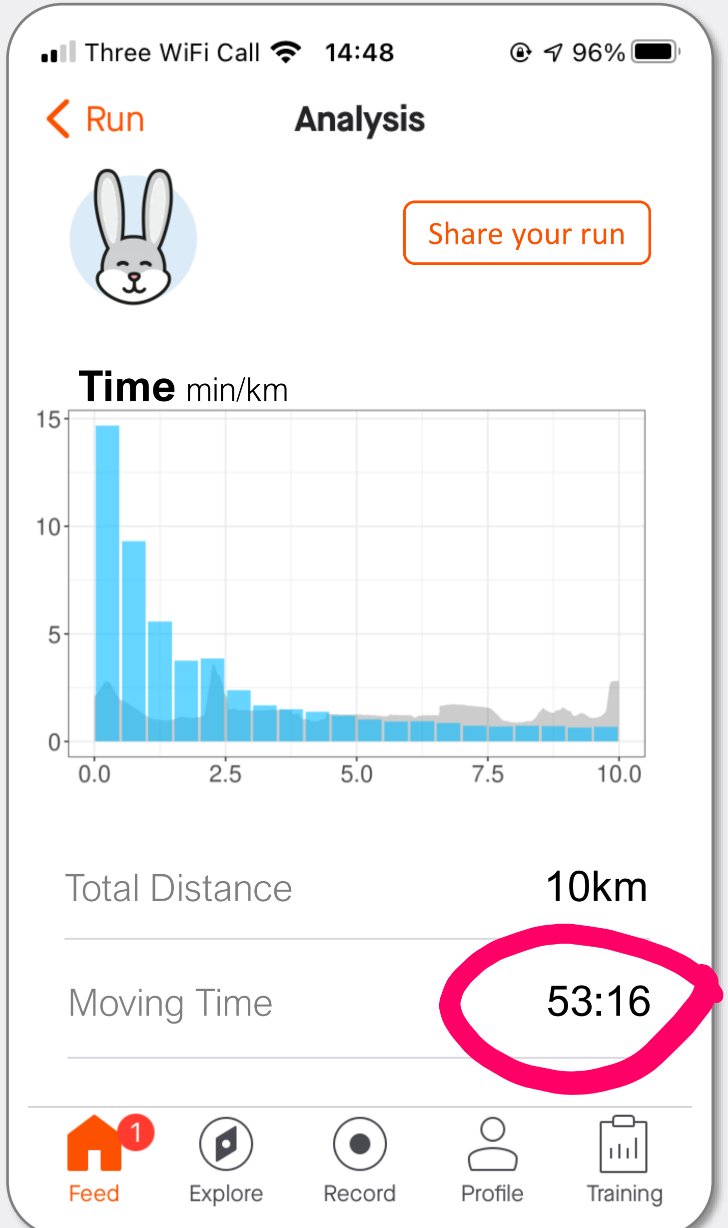
Advanced data structures

SECTION 7.1

Aggregate analysis

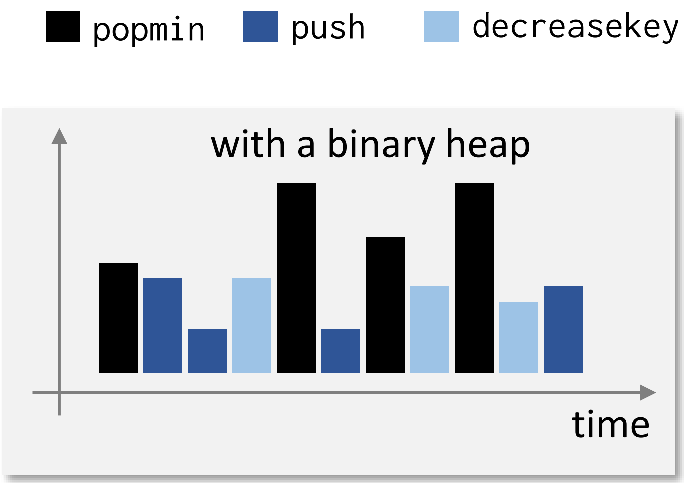
You’ve heard the moral: “slow and steady wins the race”.

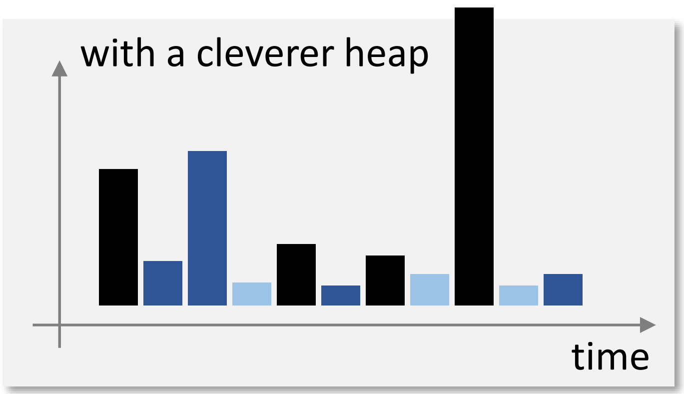




My version: “whoever finishes the race fastest wins the race”.

Running time of each operation,
in a run of Dijkstra's algorithm





total time = $O(V) \times c_{\text{popmin}}$
$+O(E) \times c_{\text{push/dec.key}}$

Don't worry about the worst-case cost of each individual operation.

Worry about the worst-case aggregate cost of a *sequence* of operations.

Advanced data structures involve a clever design tradeoff, to make sequences of operations cheaper:

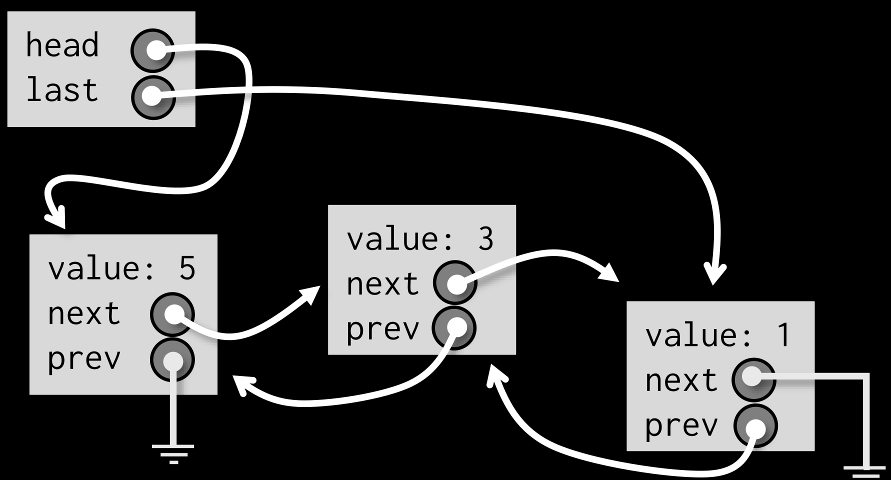
- individual operations are usually cheap, but occasionally expensive
- the worst-case aggregate cost of a sequence of m operations is cheaper than m times the worst-case of a single operation

DOUBLY-LINKED LIST

`x = DoublyLinkedList(⋯)` n items

`x[`i`]` $O(n)$

`x.append(·)` $O(1)$



PYTHON LIST

`x = [⋯]` n items

`x[`i`]` $O(1)$

`x.append(·)` $O(1)$ usually, sometimes $O(n)$

but $m \times$ append(·) is ALWAYS $m \times O(1)$

To design advanced data structures, we need to be able to reason about aggregate costs. How?

- Just be clever and work hard
- Use an accounting trick called *amortized costs*

x.append(·) $O(1)$ usually, sometimes $O(n)$

but $m \times$ append(·)

is ALWAYS $m \times O(1)$

SECTION 7.2, 7.3

Amortized costs

```
class MinList<T>:
    def append(T value):
        # append a new value
    def flush():
        # empty the list
    def foreach(f):
        # do f(x) for each item
    def T min():
        # return the smallest
        # (without removing it)
```

append O(1)
min O(n)
n = # items.

app min min

min is O(n)
worst case.

append still O(1)
min is now O(1)
But isn't there lots of
wasted work?

Stage 0

- Use a linked list
- `min` iterates over the entire list

Stage 1

- Use a linked list
- `min` caches its result, so that next time it only needs to iterate over newer values

Stage 2

- Use a linked list
- Store the current minimum, and update it on every append

Stage 3

- `min` caches its result, the same as Stage 1
- ... but we argue it's just as good as Stage 2