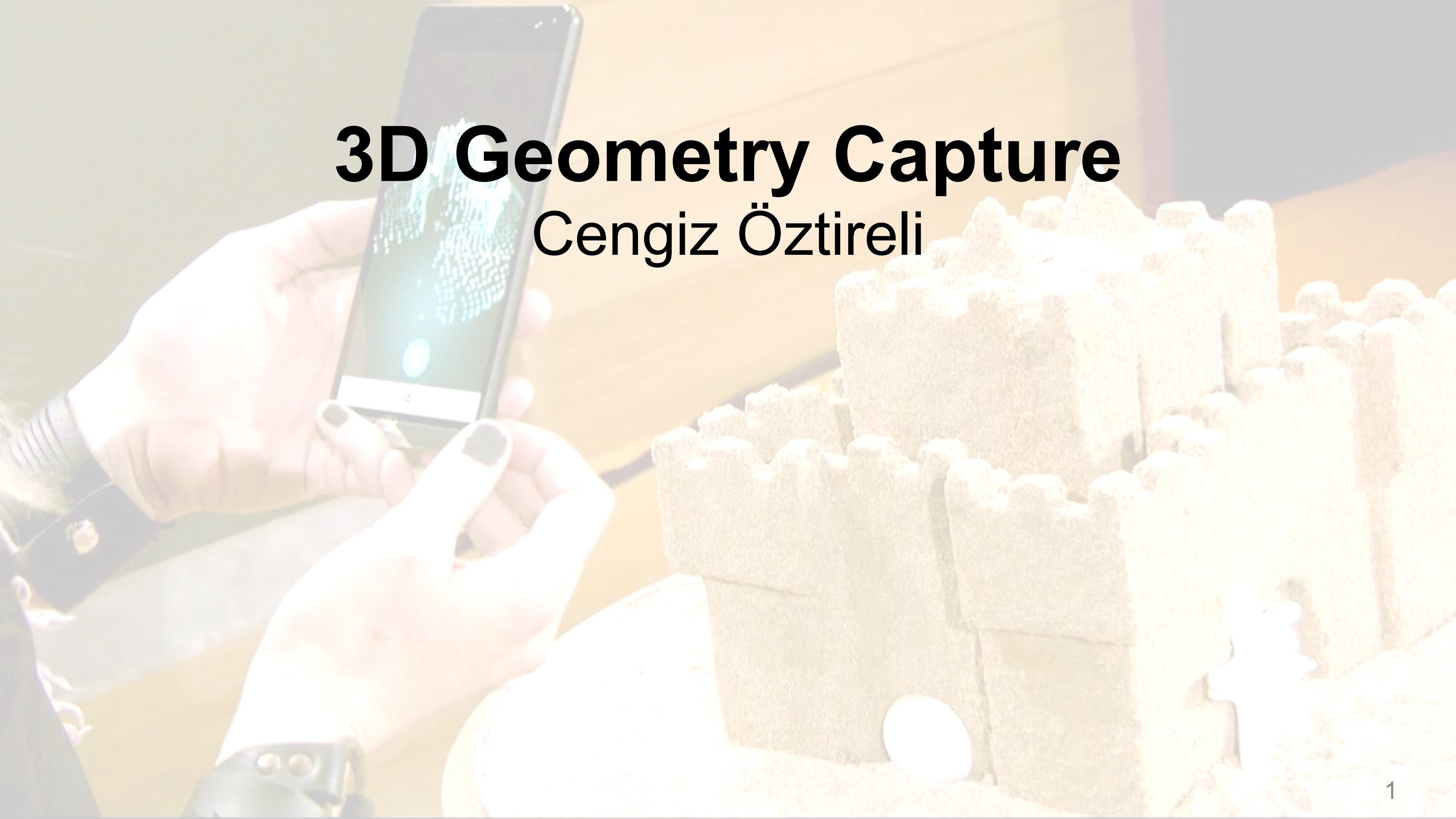


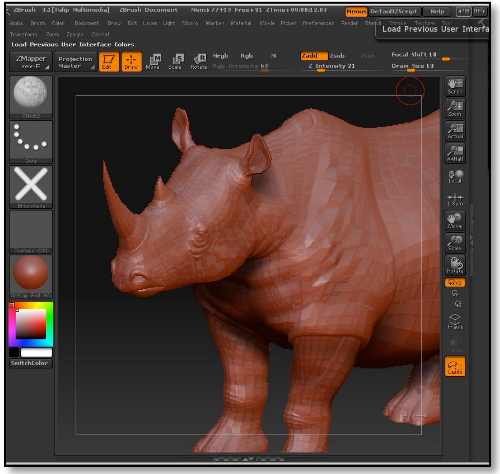
3D Geometry Capture

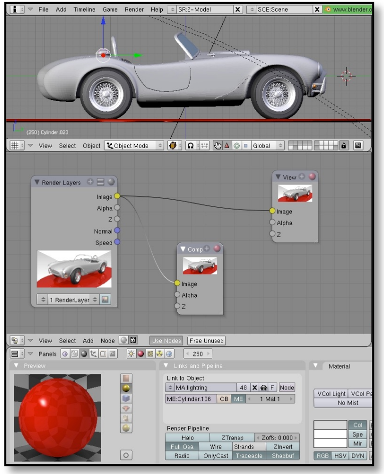
Cengiz Öztireli

Sources of Geometry

Acquisition from the real world

Modeling applications





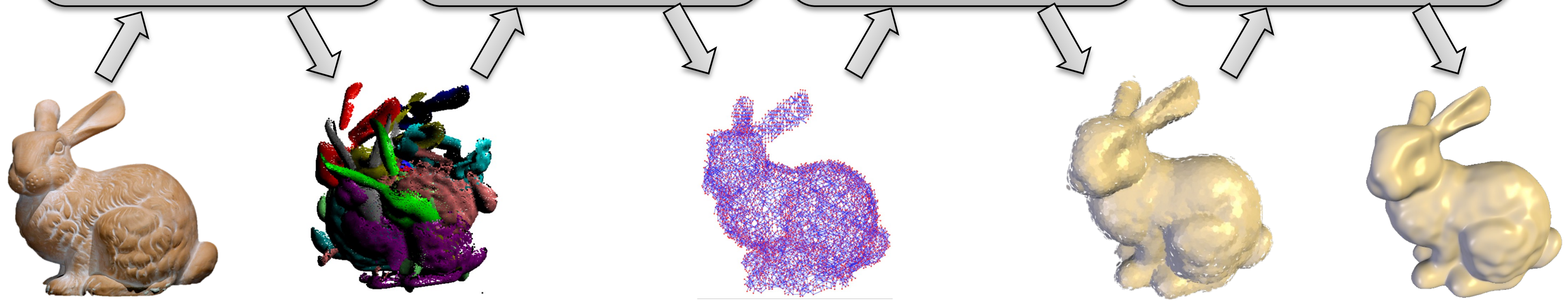

Buildings $3900
Buildings $3900
Buildings $6,000
Moving Sky $4,200
Building $3900
Buildings $3900
Building $3900
Bannister $2,450
Buildings $6,000
Waving flags $2,250
Billboard $1,313
Building Front $3,800
Tree $2,200
Tree $2,200
Bridge $3,500
Truck $5,600
DO NOT ENTER
Car $6,600
Car $7,200
Tree $2,200
Car Damaged $7,200
Car $6,000
Big Trash $14,400
NPC $22,500
Hero Character $49,000
Road Texturing $11,400
Traffic Cones $3,600
Concrete Divider: $960
Barrel $940
Trash $3,438
Road Texturing $10,800
Estimated Creation Cost: $200,000

The West Cambridge Digital Twin project

Shape Acquisition

- Digitizing real world objects

3D Scanning → Registration → Pre-processing → Reconstruction



3D Scanning

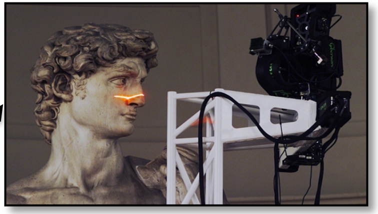
Touch Probes

+ Precise
- Small objects

Optical Scanning

+ Fast
- Glossy objects

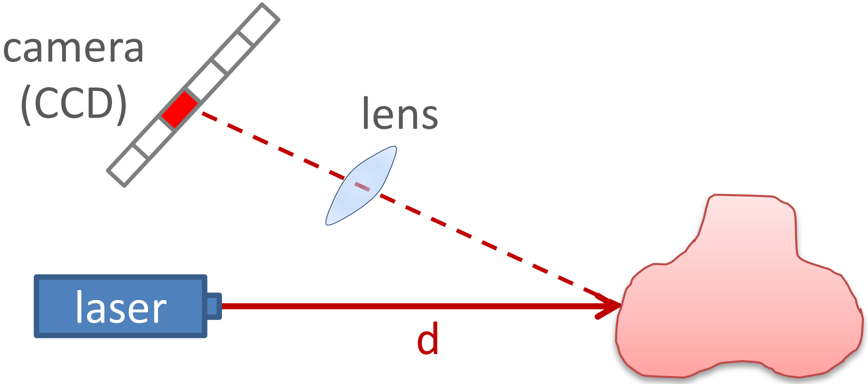
Active

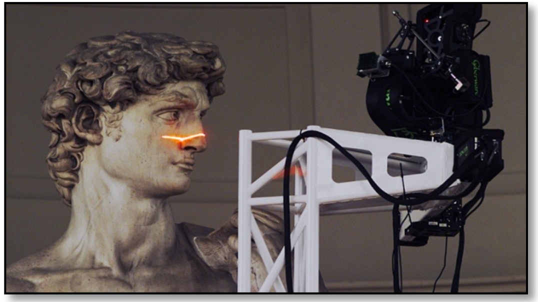


Passive

Active Systems

- Triangulation Laser
  - Laser beam and camera
  - Laser dot is photographed
  - The location of the dot in the image allows triangulation: we get the distance to the object



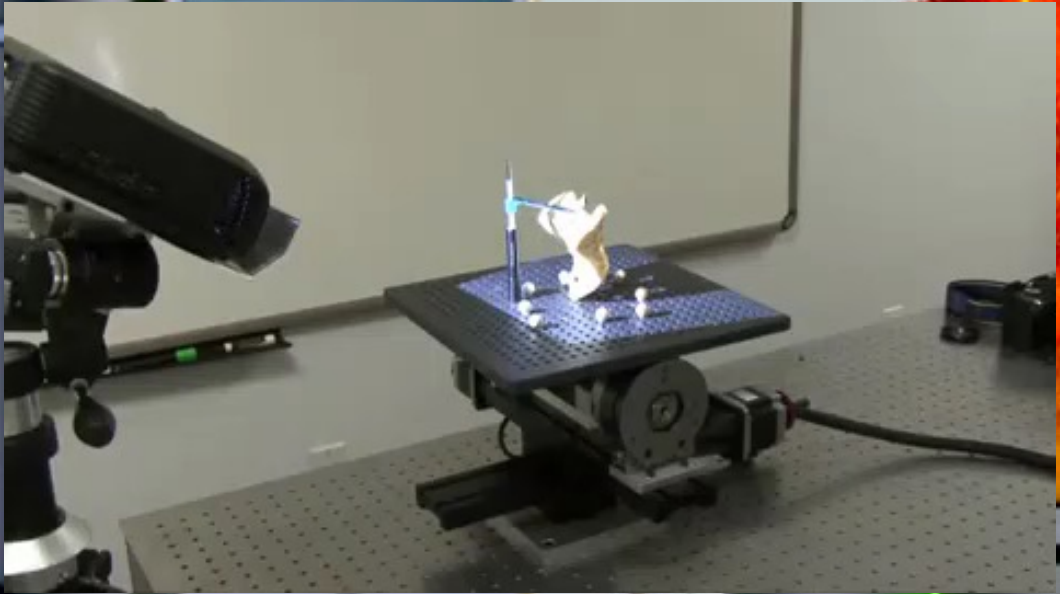


Active Systems

Structured light

Active Systems

Structured light



Active Systems

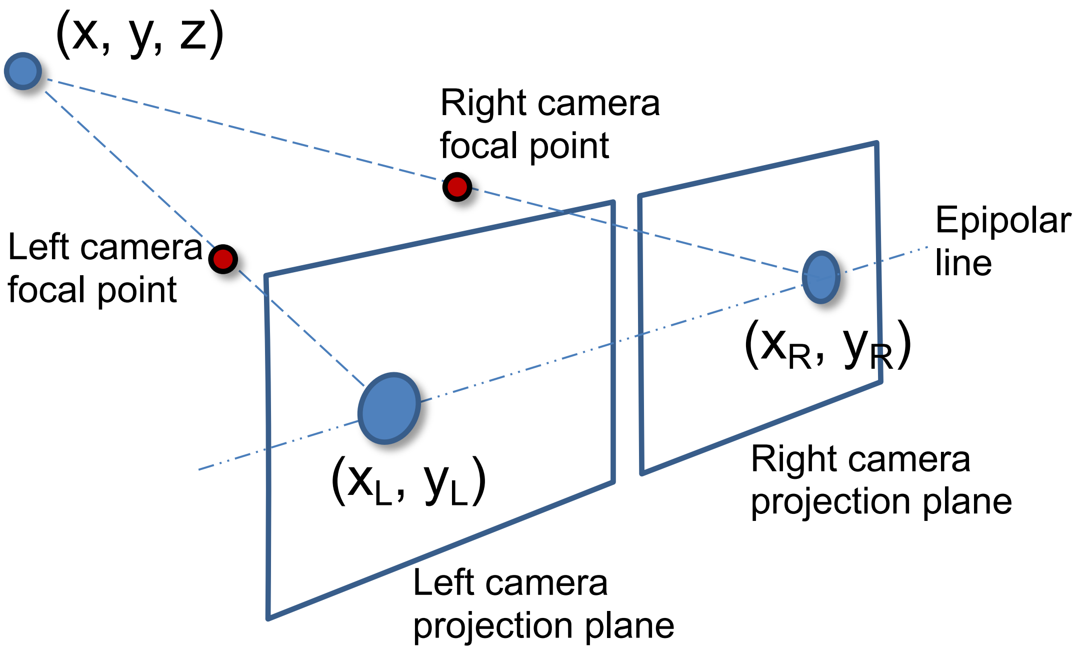
- Structured light
  - Pattern of visible or **infrared** light is projected onto the object
  - The distortion of the pattern (recorded by the camera) provides geometric information
  - Very fast – 2D pattern at once
  - Complex distance calculation → prone to noise

Active Systems

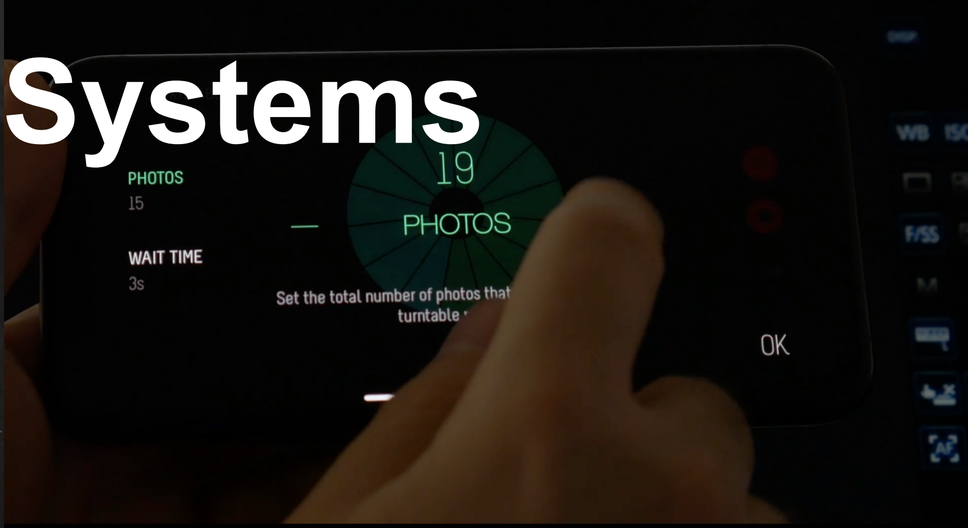
- LIDAR
  - Measures the time it takes the laser beam to hit the object and come back

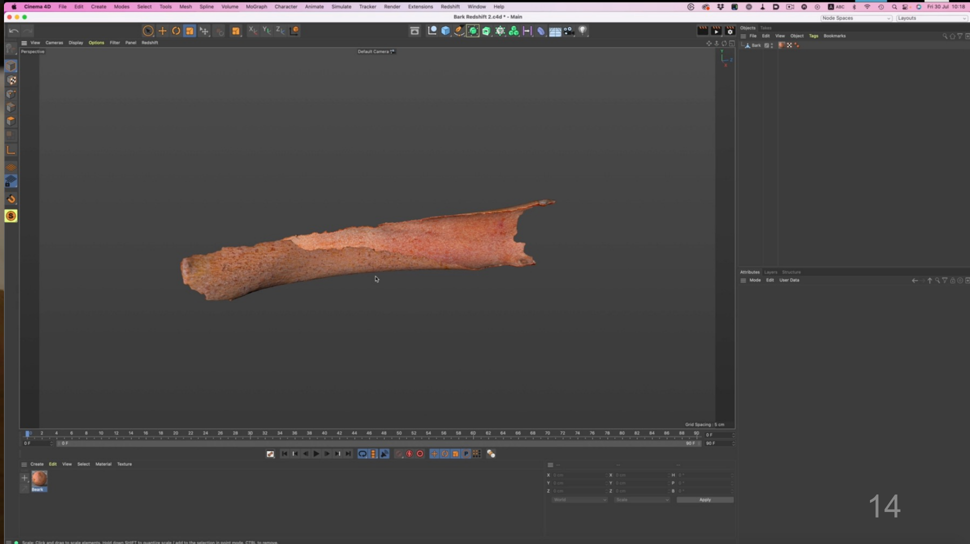
Passive Systems

Multi-view Stereo



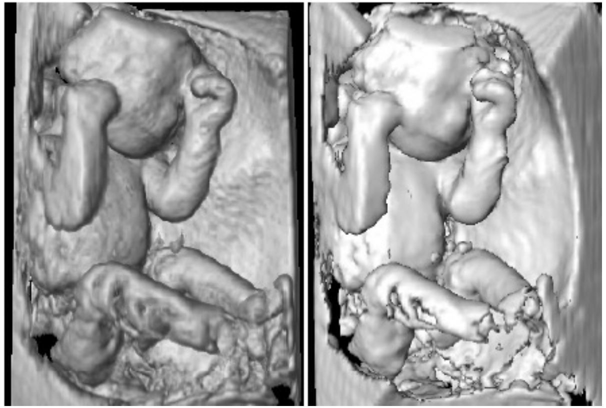
Passive Systems

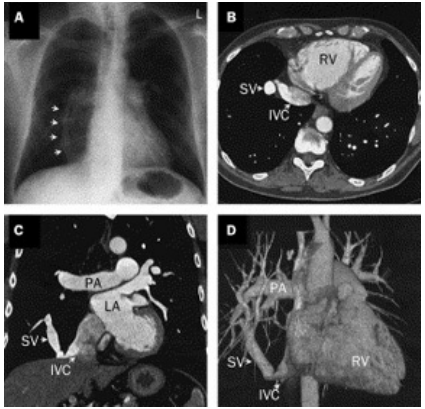


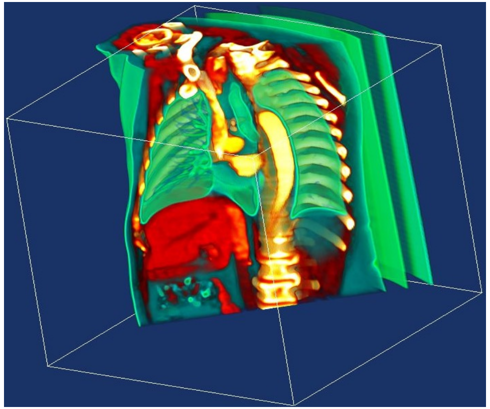


3D Imaging

- Ultrasound, CT, MRI
- Discrete volume of density data
- First need to segment the desired object (contouring)





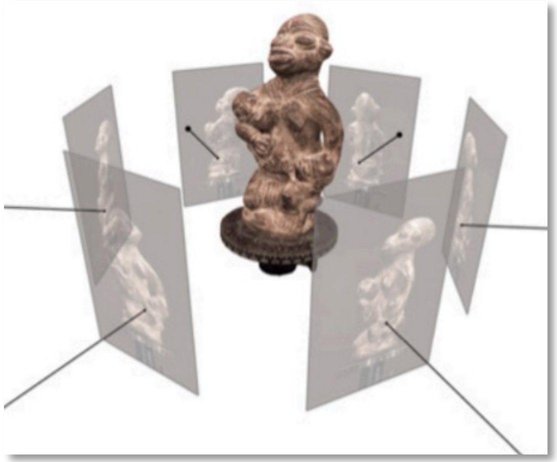


3D Scanning

- Challenges

Noise, outliers, irregularity

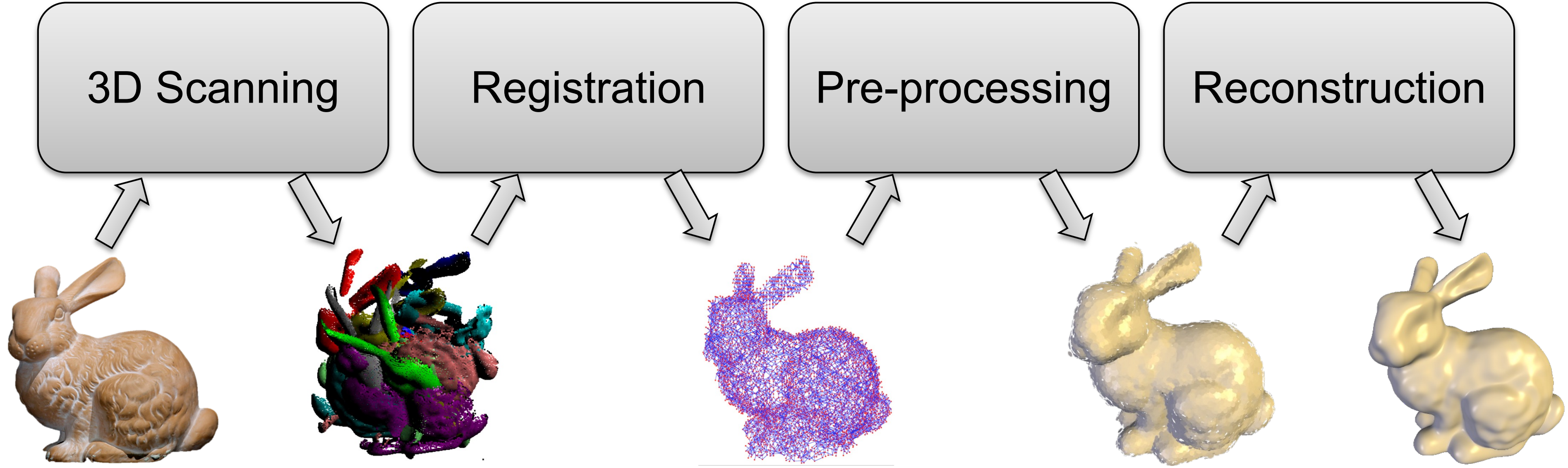
Incompleteness



Inconsistency

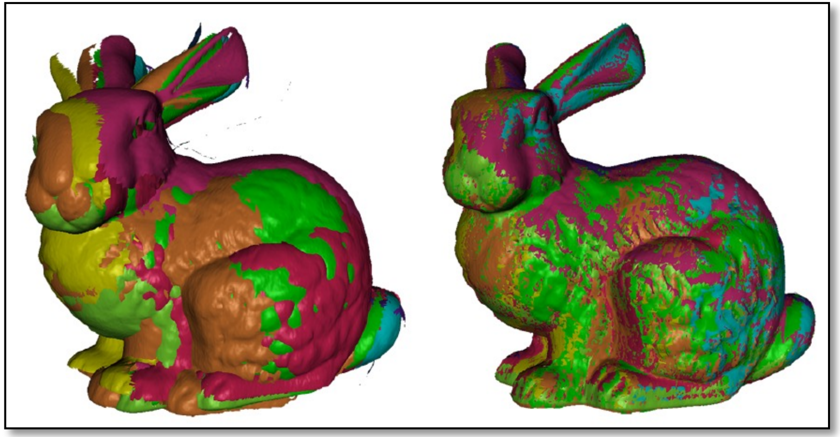
Shape Acquisition

- Digitizing real world objects

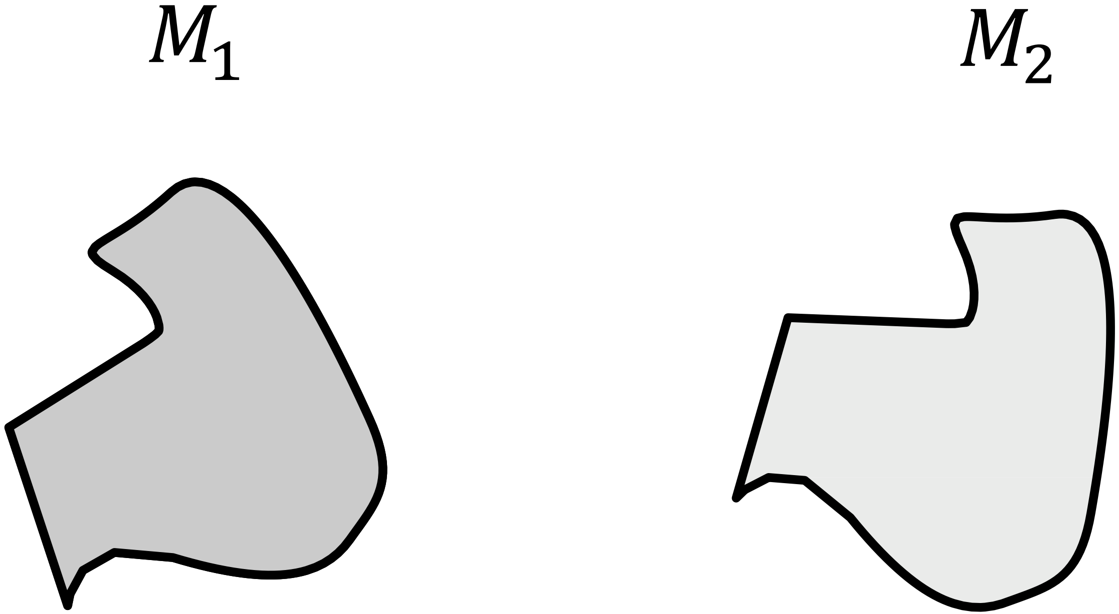


Registration

- Bringing scans into a common coordinate frame

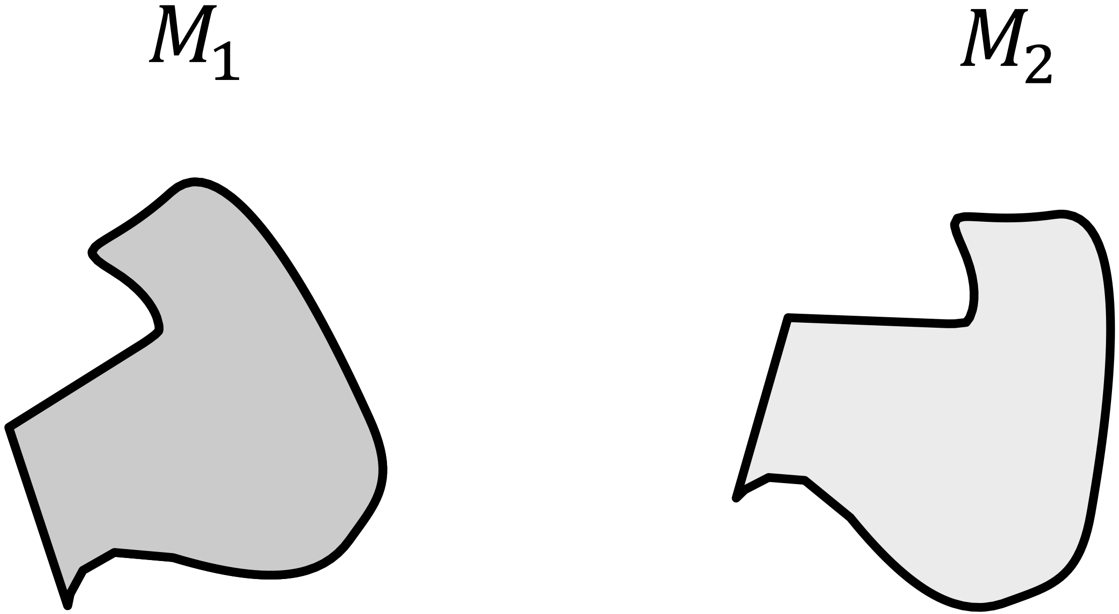


Registration



$$M_1 \approx T(M_2), T\text{: translation + rotation}$$

Registration



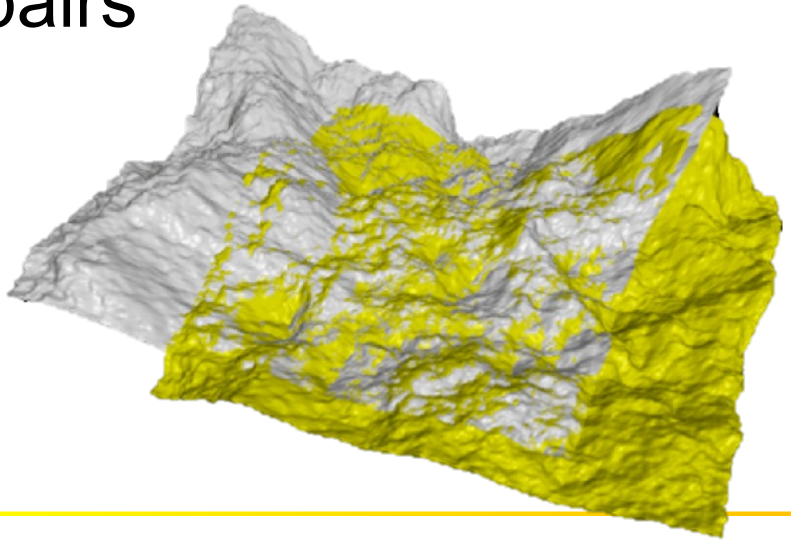
$$M_1 \approx T_2(M_2) \approx \cdots \approx T_n(M_n)$$

Registration

- How many points are needed
  to define a unique rigid transformation?
- The first problem is finding pairs

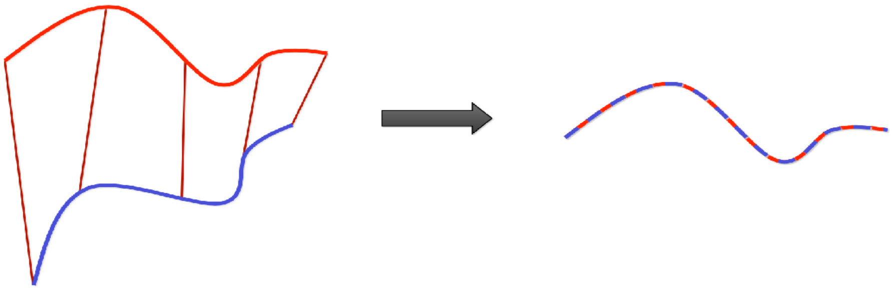
$$\mathbf{p}_1 \rightarrow \mathbf{q}_1$$
$$\mathbf{p}_2 \rightarrow \mathbf{q}_2$$
$$\mathbf{p}_3 \rightarrow \mathbf{q}_3$$

$$R\mathbf{p}_i + t \approx \mathbf{q}_i$$



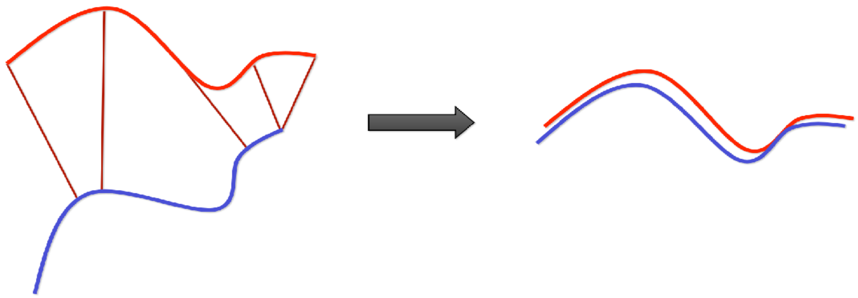
Registration

- ICP: Iterative Closest Point
- Idea: Iterate
  - (1) Find correspondences
  - (2) Use them to find a transformation



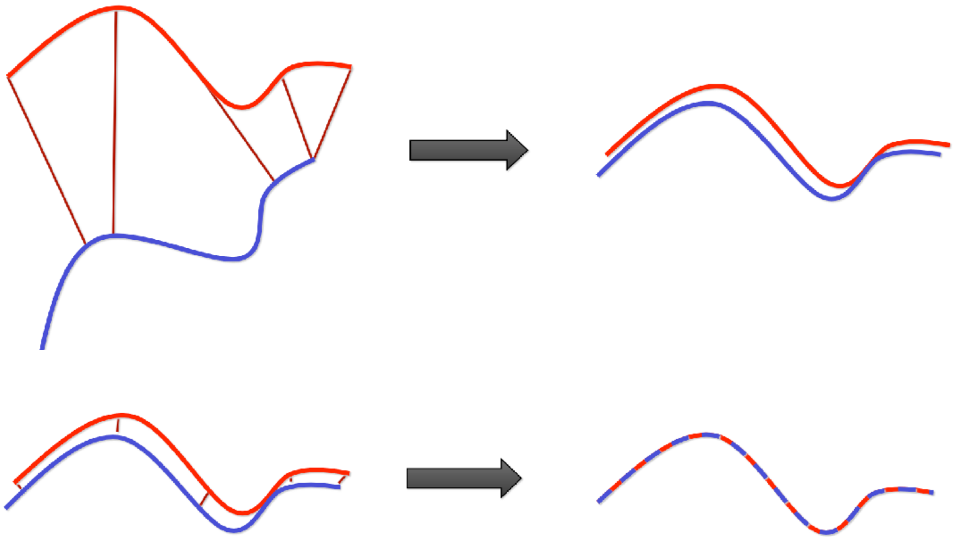
Registration

- ICP: Iterative Closest Point
- Intuition:
  - With the right correspondences, problem solved
  - If you don't have the right ones, can still make progress



Registration

- ICP: Iterative Closest Point



Registration

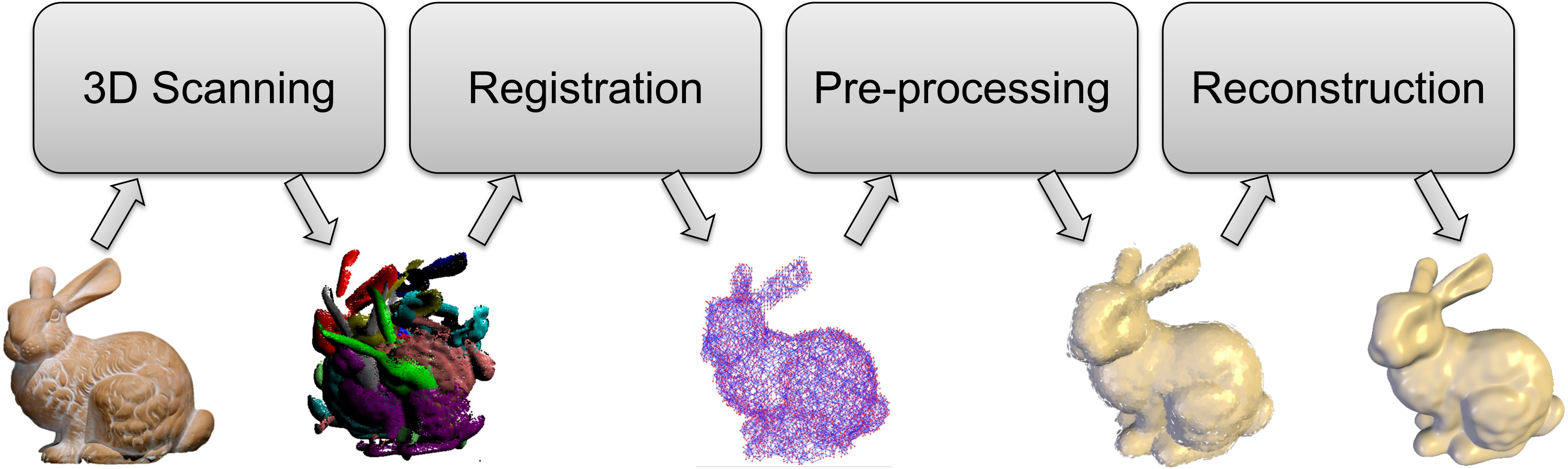
- ICP: Iterative Closest Point -- algorithm
  - Select (e.g., 1000) random points
  - Match each to closest point on other scan
  - Reject pairs with distance too big
  - Construct error function:

$$E := \sum_i (R\mathbf{p}_i + t - \mathbf{q}_i)^2$$

  - Minimize
    - closed form solution in: http://dl.acm.org/citation.cfm?id=250160

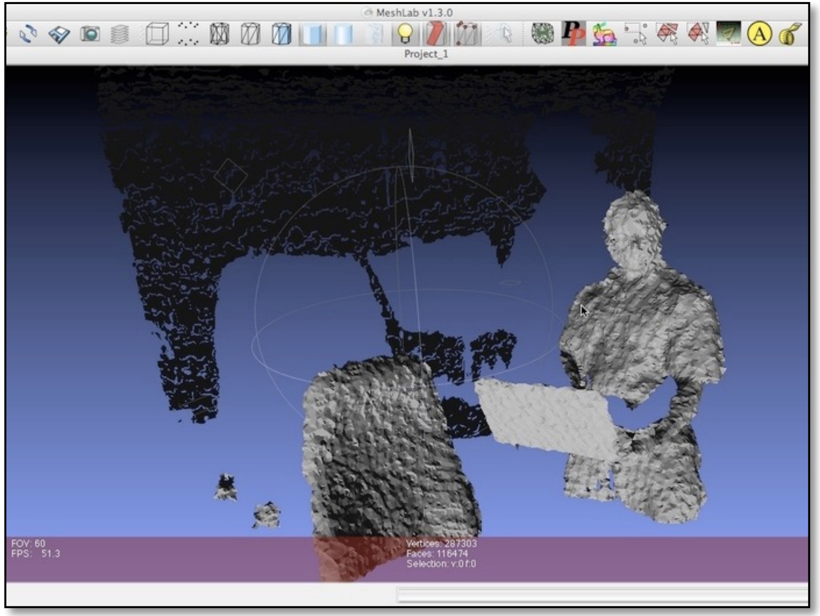
Shape Acquisition

- Digitizing real world objects



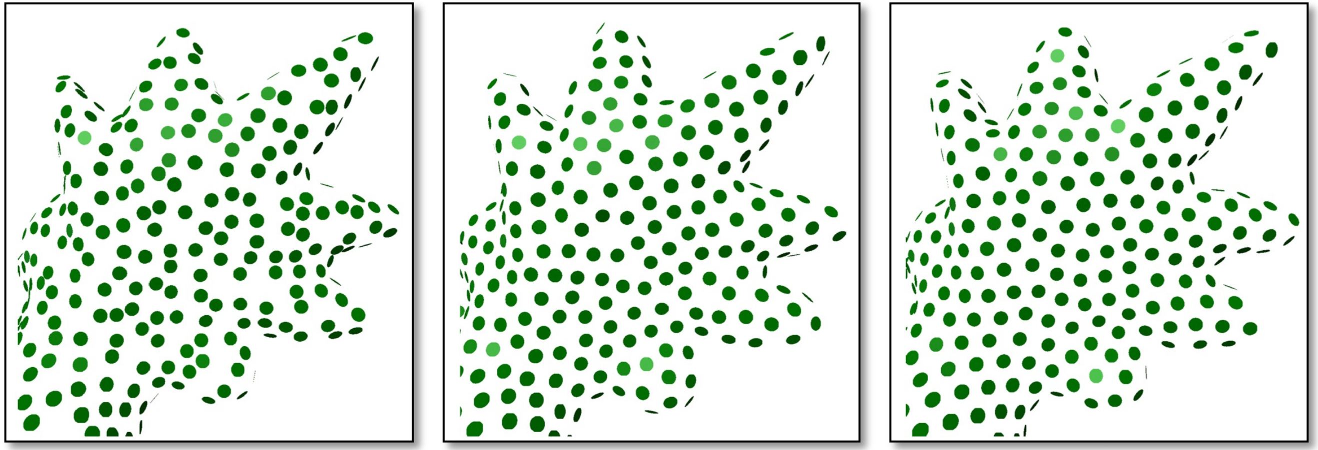
Pre-processing

- Cleaning, repairing, resampling



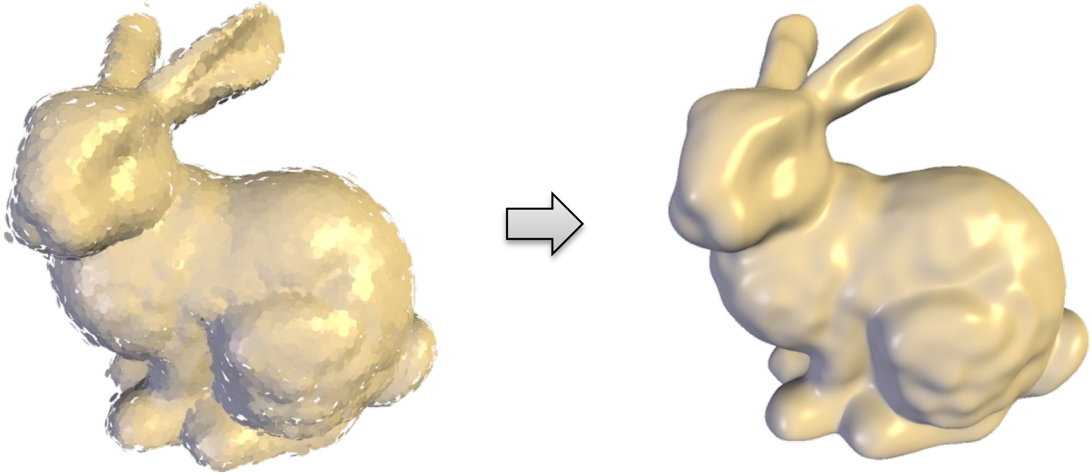
Pre-processing

- Sampling for accurate reconstructions



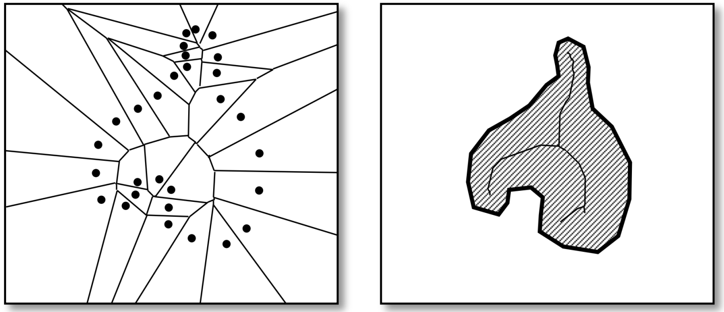
Reconstruction

- Mathematical representation for a shape



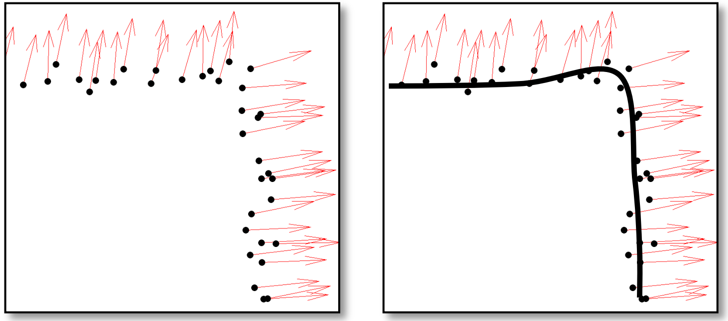
Reconstruction

Connect-the-points Methods



+ Theoretical error bounds
– Expensive
– Not robust to noise

Approximation-based Methods



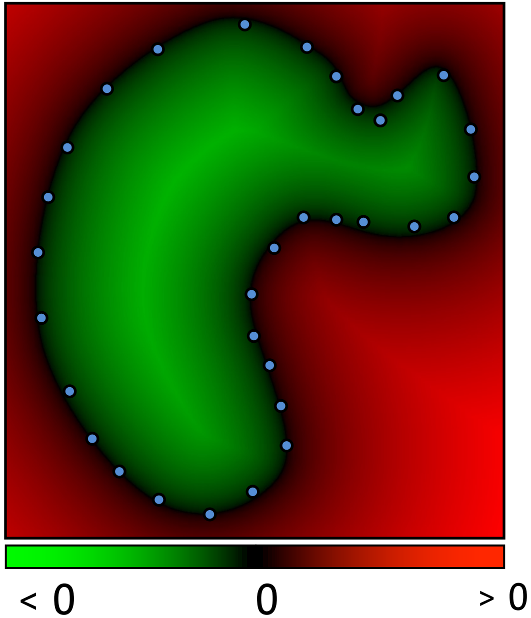
+ Efficient to compute
+ Robust to noise
– No theoretical error bounds

Reconstruction

- Approximating an implicit function

$$f : \mathbb{R}^3 \to \mathbb{R}$$

with value > 0 outside the shape and < 0 inside



Reconstruction

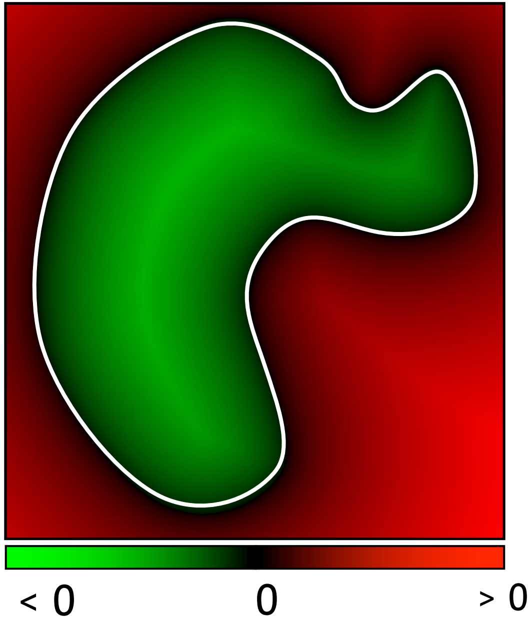
- Approximating an implicit function

$$f : \mathbb{R}^3 \to \mathbb{R}$$

with value > 0 outside
the shape and < 0 inside

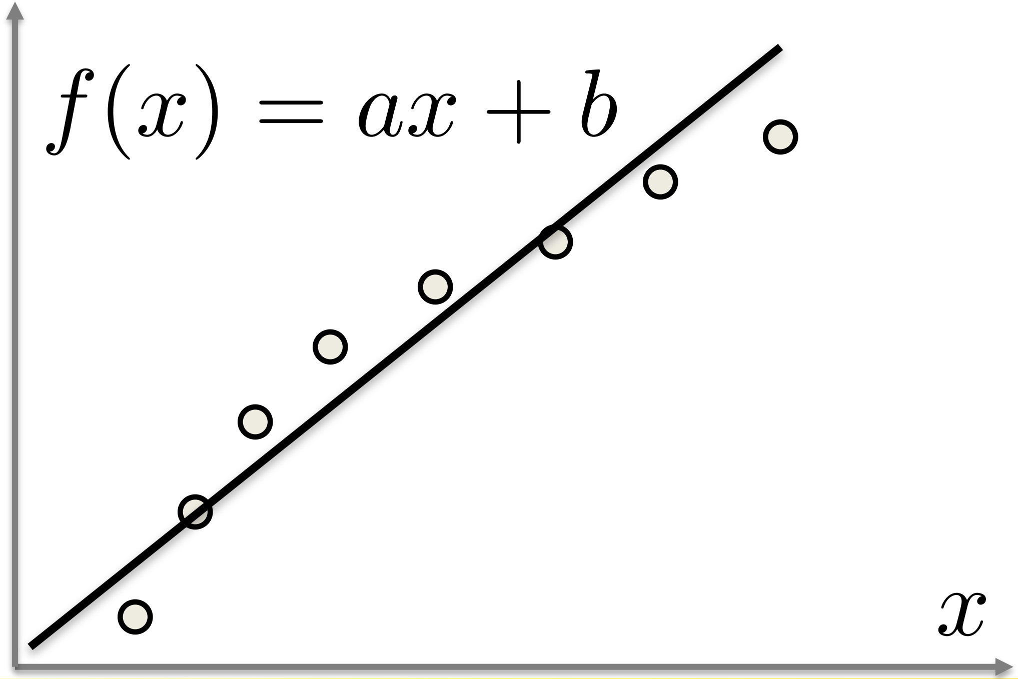
$$\{\mathbf{x} : f(\mathbf{x}) = 0\}$$

extract zero set



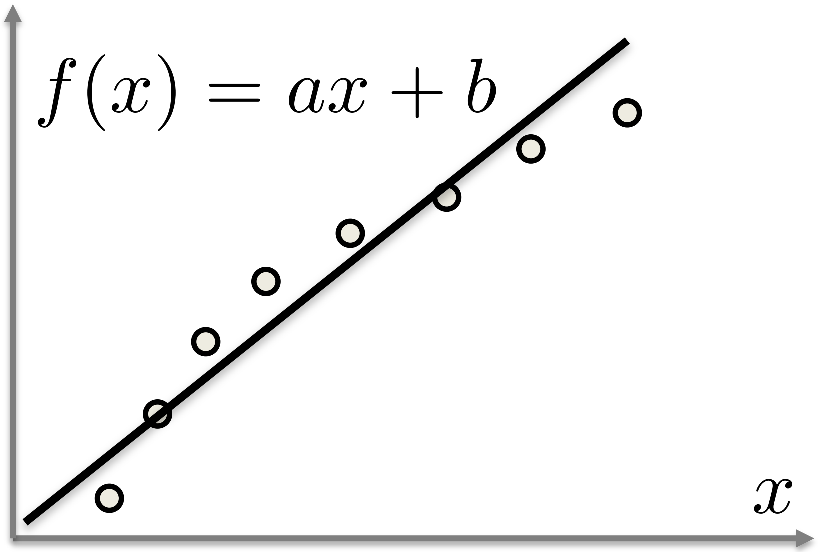
Least Squares

- Problem



Least Squares

- Problem



$$\min_{a,b} \sum_{i=1}^{n} (f(x_i) - y_i)^2$$

$$\min_{a,b} \sum_{i=1}^{n} (ax_i + b - y_i)^2$$

Least Squares

- Multi-dimensional problem

$$f(\mathbf{x}) : \mathbb{R}^d \to \mathbb{R} \qquad \min_{f \in \Pi_m^d} \sum_i \left(f(\mathbf{x}_i) - f_i\right)^2$$

Π_m^d : polynomials of degree m in d dimensions

$$f(\mathbf{x}) = \mathbf{b}(\mathbf{x})^T \mathbf{c}$$

$$m = 2, d = 2 \qquad \mathbf{b}(\mathbf{x}) = \left[1 \; x \; y \; x^2 \; y^2 \; xy\right]^T$$

$$f(\mathbf{x}) = c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 y^2 + c_5 xy$$

Least Squares

- Multi-dimensional problem

$$f(\mathbf{x}) : \mathbb{R}^d \rightarrow \mathbb{R} \qquad \min_{f \in \Pi_m^d} \sum_i \left(f(\mathbf{x}_i) - f_i\right)^2$$

$$f(\mathbf{x}) = \mathbf{b}(\mathbf{x})^T \mathbf{c}$$

$$\min_{\mathbf{c}} E(\mathbf{c}) \qquad E(\mathbf{c}) = \sum_i \left(\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} - f_i\right)^2$$

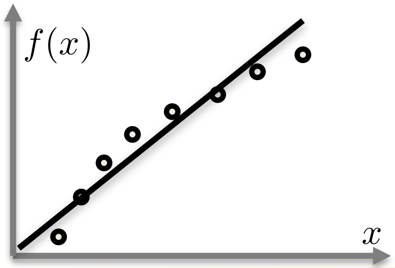
Least Squares

- Multi-dimensional problem

$$\min_{\mathbf{c}} E(\mathbf{c}) \qquad E(\mathbf{c}) = \sum_i \left(\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} - f_i\right)^2$$

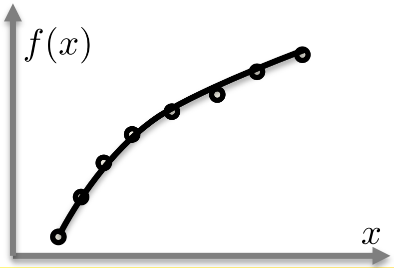
$$m = 1, d = 1$$

$$E(\mathbf{c}) = \sum_i \left(c_0 + c_1 x_i - f_i\right)^2$$



$$m = 2, d = 1$$

$$E(\mathbf{c}) = \sum_i \left(c_0 + c_1 x_i + c_2 x_i^2 - f_i\right)^2$$



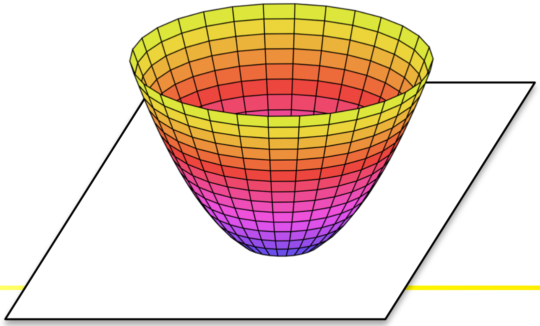
Least Squares

- Multi-dimensional problem

$$\min_{\mathbf{c}} E(\mathbf{c}) \qquad E(\mathbf{c}) = \sum_i \left(\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} - f_i\right)^2$$

$$m = 2, d = 2$$

$$E(\mathbf{c}) = \sum_i \left(c_0 + c_1 x + c_2 y + c_3 x^2 + c_4 y^2 + c_5 xy - f_i\right)^2$$



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Least Squares

- Solution of the multi-dimensional problem

$$\min_{\mathbf{c}} E(\mathbf{c}) \qquad E(\mathbf{c}) = \sum_i \left(\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} - f_i\right)^2 \quad \mathbf{b}(\mathbf{x}_i) = [b_1(\mathbf{x}_i) \cdots b_m(\mathbf{x}_i)]^T$$

$$\frac{\partial E(\mathbf{c})}{\partial c_k} = \sum_i 2b_k(\mathbf{x}_i)\left[\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} - f_i\right] = 0$$

$$\frac{\partial E(\mathbf{c})}{\partial \mathbf{c}} = 2\sum_i \mathbf{b}(\mathbf{x}_i)\left[\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} - f_i\right] = 0$$

$$\sum_i \mathbf{b}(\mathbf{x}_i)\mathbf{b}(\mathbf{x}_i)^T\mathbf{c} = \sum_i \mathbf{b}(\mathbf{x}_i) f_i$$

$$\boxed{\mathbf{c} = \left[\sum_i \mathbf{b}(\mathbf{x}_i)\mathbf{b}(\mathbf{x}_i)^T\right]^{-1} \sum_i \mathbf{b}(\mathbf{x}_i) f_i}$$

Least Squares

- Solution of the multi-dimensional problem

Example

$$m = 2, d = 1 \quad E(\mathbf{c}) = \sum_i \left(c_0 + c_1 x + c_2 x^2 - f_i\right)^2$$

$$\sum_i \begin{bmatrix} 1 & x_i & x_i^2 \\ x_i & x_i^2 & x_i^3 \\ x_i^2 & x_i^3 & x_i^4 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \end{bmatrix} = \sum_i \begin{bmatrix} 1 \\ x_i \\ x_i^2 \end{bmatrix} f_i$$

Weighted Least Squares

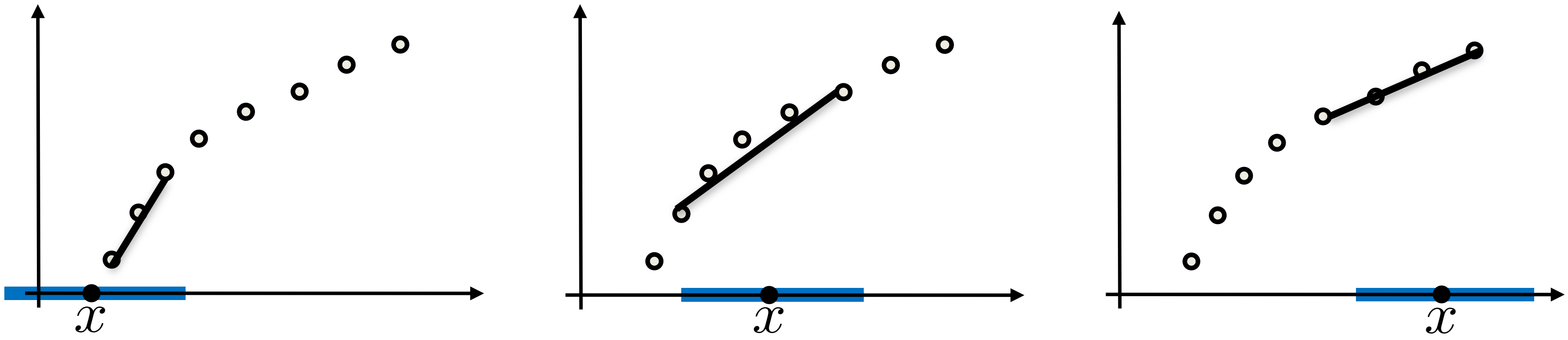
- Multiply the terms with given weights

LS $\min_{\mathbf{c}} E(\mathbf{c}) \qquad E(\mathbf{c}) = \sum_i \left(\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} - f_i\right)^2$

WLS $\min_{\mathbf{c}} E(\mathbf{c}) \qquad E(\mathbf{c}) = \sum_i \left(\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} - f_i\right)^2 w_i$

Moving Least Squares

- Idea: make the weights local



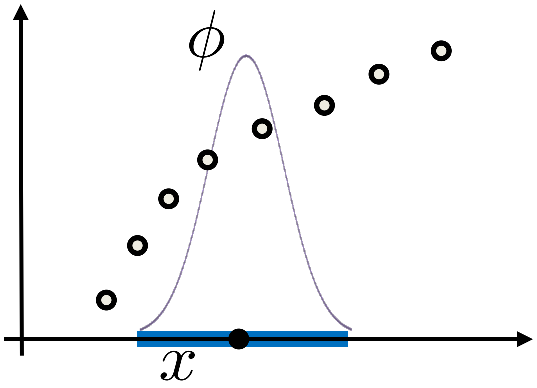
Moving Least Squares

- Idea: make the weights local

$$f(\mathbf{x}) = \min_{\boxed{f_{\mathbf{x}}} \in \Pi_m^d} \sum_i \boxed{\phi(||\mathbf{x} - \mathbf{x}_i||)} (\boxed{f_{\mathbf{x}}}(\mathbf{x}_i) - f_i)^2$$

Local approximation

Weights depend on x



Moving Least Squares

- Idea: make the weights local

$$\mathbf{c}(\mathbf{x}) = \operatorname{argmin}_{\mathbf{c}} E_{\mathbf{x}}(\mathbf{c}) = \sum_i \phi(||\mathbf{x} - \mathbf{x}_i||) \left(\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} - f_i\right)^2$$

$$f(\mathbf{x}) = \mathbf{b}(\mathbf{x})^T \mathbf{c}(\mathbf{x})$$

In comparison, LS:

$$\mathbf{c} = \operatorname{argmin}_{\mathbf{c}} E(\mathbf{c}) = \sum_i \left(\mathbf{b}(\mathbf{x}_i)^T \mathbf{c} - f_i\right)^2$$

$$f(\mathbf{x}) = \mathbf{b}(\mathbf{x})^T \mathbf{c}$$

Moving Least Squares

- Local solution

$$\mathbf{c}(\mathbf{x}) = \left[\sum_i \phi_i(\mathbf{x})\mathbf{b}(\mathbf{x}_i)\mathbf{b}(\mathbf{x}_i)^T\right]^{-1} \sum_i \phi_i(\mathbf{x})\mathbf{b}(\mathbf{x}_i) f_i$$

$$\phi_i(\mathbf{x}) = \phi(||\mathbf{x} - \mathbf{x}_i||)$$

$$f(\mathbf{x}) = \mathbf{b}(\mathbf{x})^T \mathbf{c}(\mathbf{x})$$

Moving Least Squares

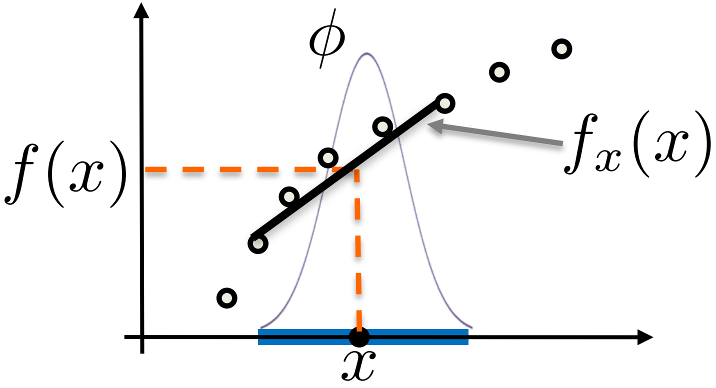
- Local solution

Example $m = 1, d = 1$

$$\min_{c_0, c_1} \sum_i \phi_i(x) \left(c_0 + c_1 x_i - f_i\right)^2$$

$$f_x(x) = c_0 + c_1 x$$

$$f(x) = f_x(x)$$



Implicit MLS Surfaces

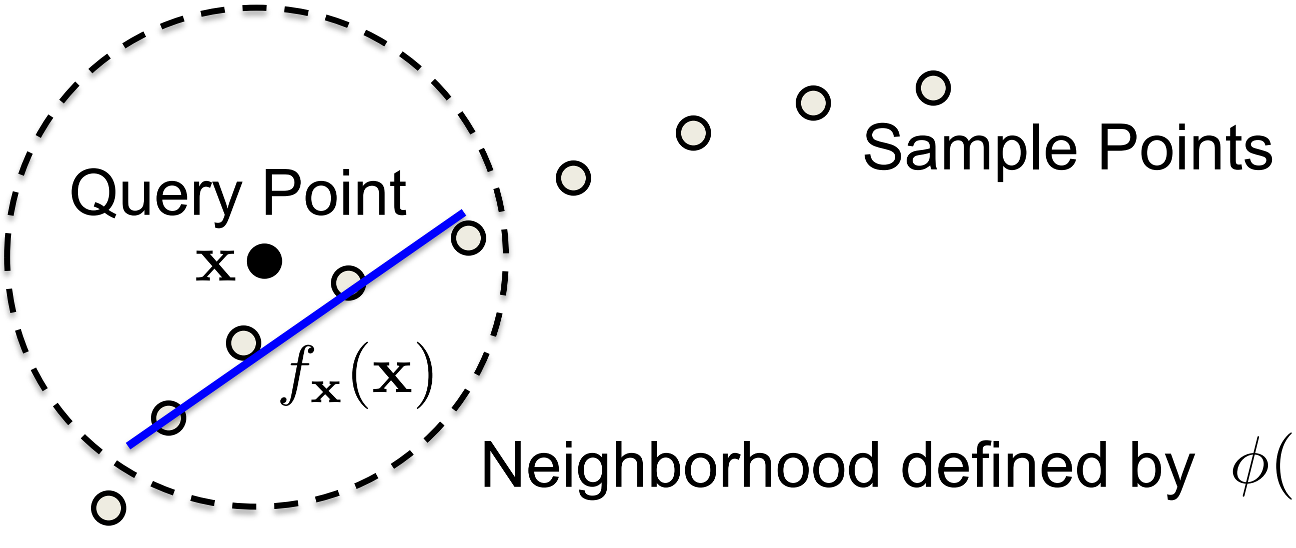
- Basic problem
  - Given sample points & attributes
  - Compute a function

$$f(\mathbf{x}) : \mathbb{R}^2 \text{ or } \mathbb{R}^3 \rightarrow \mathbb{R}$$

  - such that the curve/surface is given by

$$\mathcal{S} = \{\mathbf{x} | f(\mathbf{x}) = 0,\ \nabla f(\mathbf{x}) \neq \mathbf{0}\}$$

Implicit MLS Surfaces

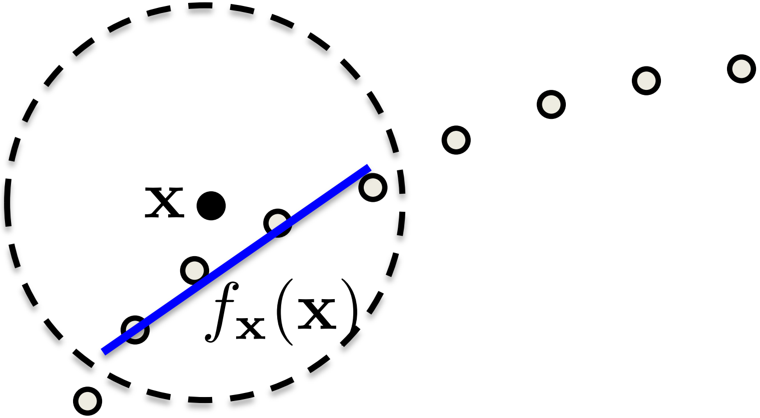


Neighborhood defined by $\phi(||\mathbf{x} - \cdot||)$

$$f(\mathbf{x}) = f_{\mathbf{x}}(\mathbf{x})$$

Implicit MLS Surfaces

Example $m = 1, d = 2$



$$f_{\mathbf{x}}(\mathbf{x}) = c_0(\mathbf{x}) + c_1(\mathbf{x})x + c_2(\mathbf{x})y$$

Implicit MLS Surfaces

How can we avoid the trivial solution

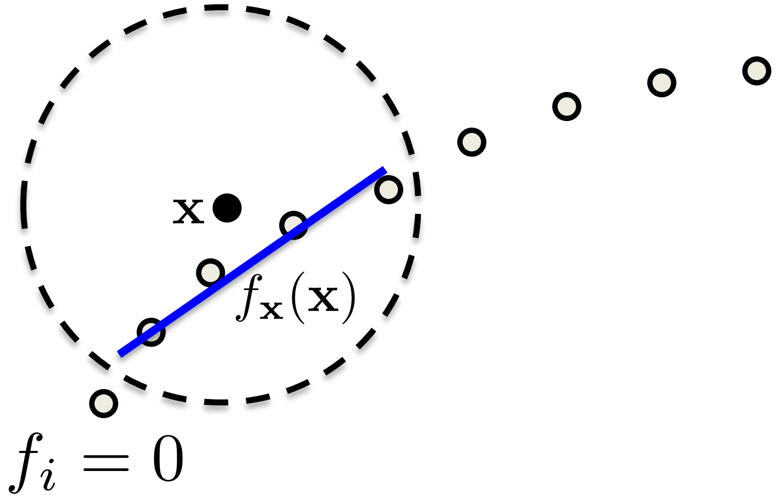
$$f(\mathbf{x}) = 0 \; \forall \mathbf{x}$$

Gradient constraints

$$||\nabla f_{\mathbf{x}}(\mathbf{x})|| = 1 \quad \nabla f(\mathbf{x}_i) = \mathbf{n}_i$$

Reproduce local functions

$$f_i(\mathbf{x}) = \mathbf{n}_i^T(\mathbf{x} - \mathbf{x}_i)$$

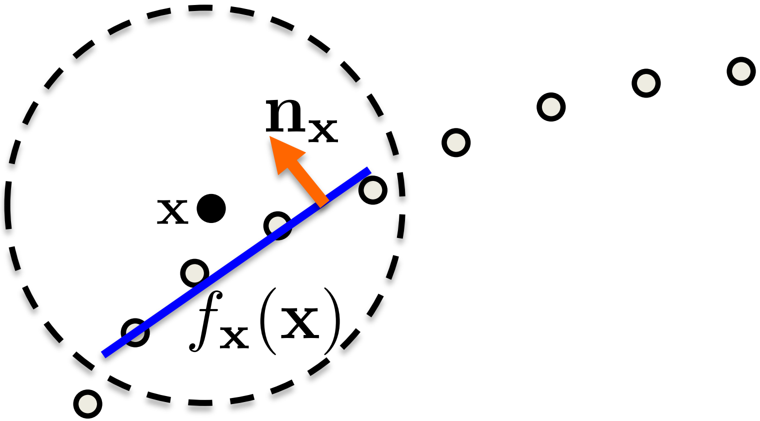


Implicit MLS Surfaces

- Example

$$m = 1, d = 2$$

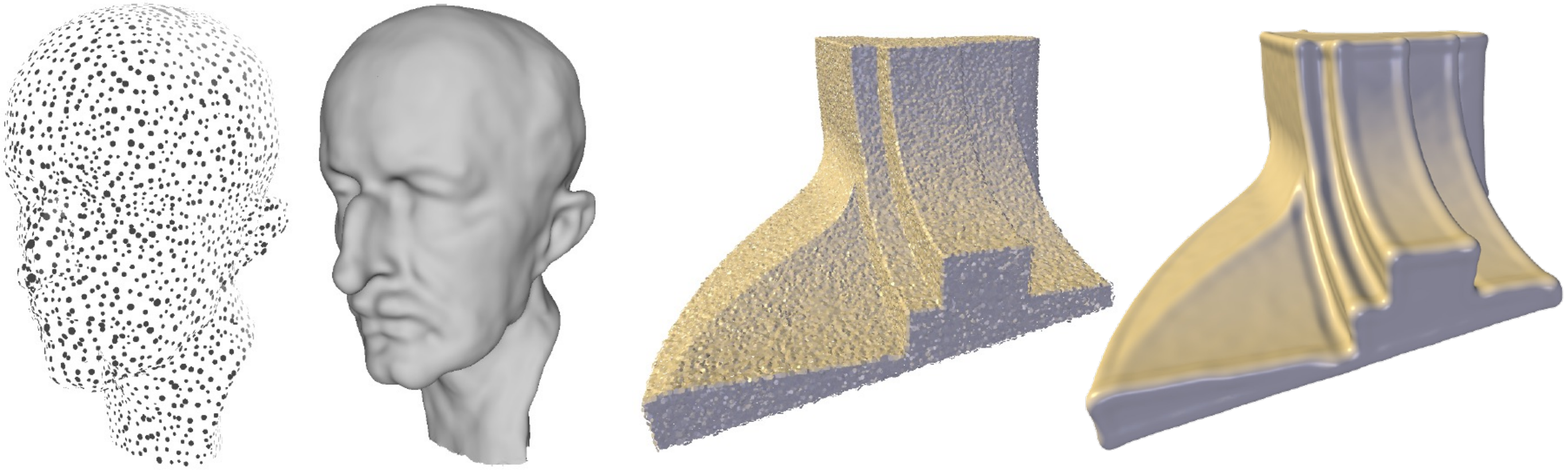
$$f_{\mathbf{x}}(\mathbf{x}) = \mathbf{n}_{\mathbf{x}}^T \mathbf{x} + o_{\mathbf{x}} \qquad ||\mathbf{n}_{\mathbf{x}}|| = 1$$



$$(\mathbf{n}_{\mathbf{x}}, o_{\mathbf{x}}) = \mathrm{argmin}_{\mathbf{n},o} \sum_i \phi_i(\mathbf{x}) \left(\mathbf{n}^T \mathbf{x}_i + o\right)^2 \qquad ||\mathbf{n}|| = 1$$

Implicit MLS Surfaces

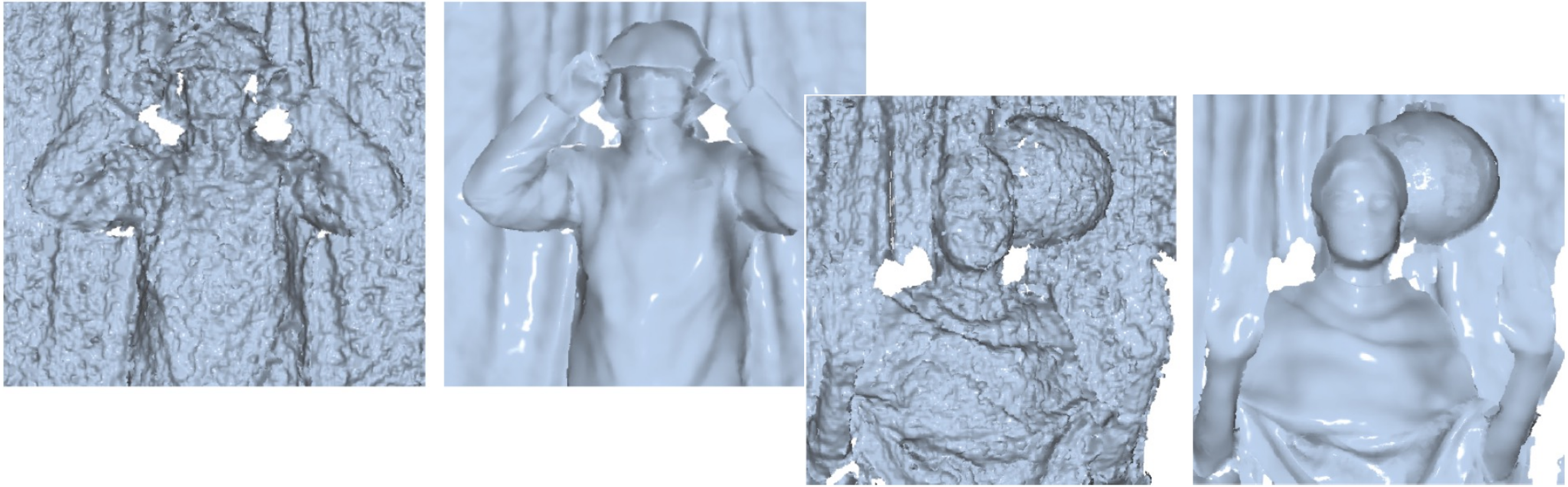
- Examples in 3D



Feature Preserving Point Set Surfaces based on Non-Linear Kernel Regression, Eurographics 2009

Implicit MLS Surfaces

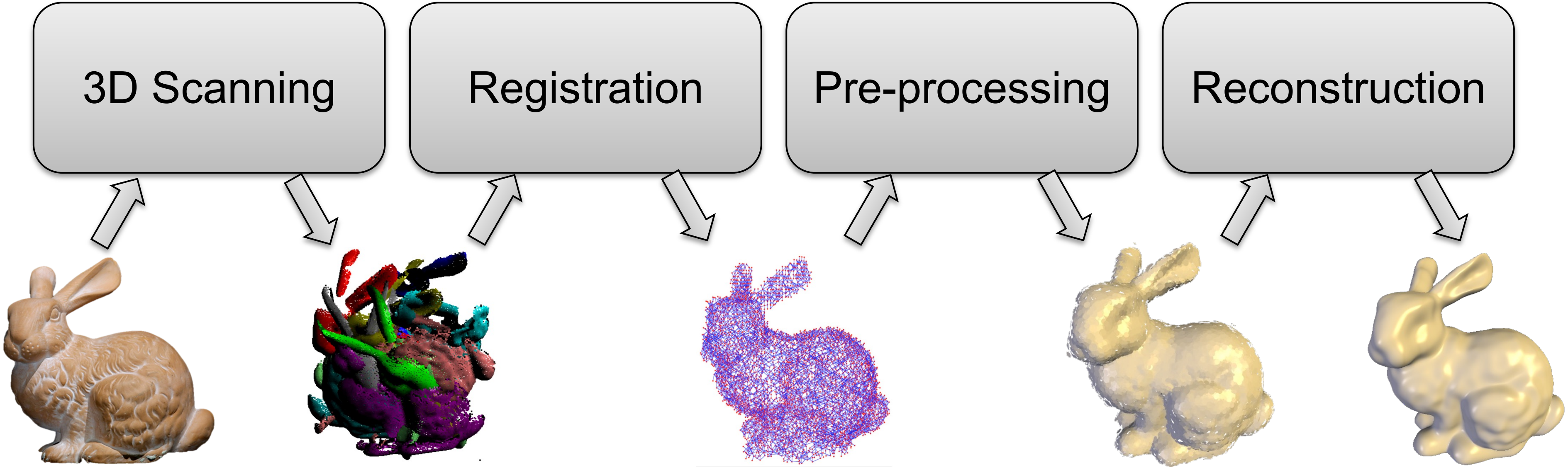
- Examples in 3D



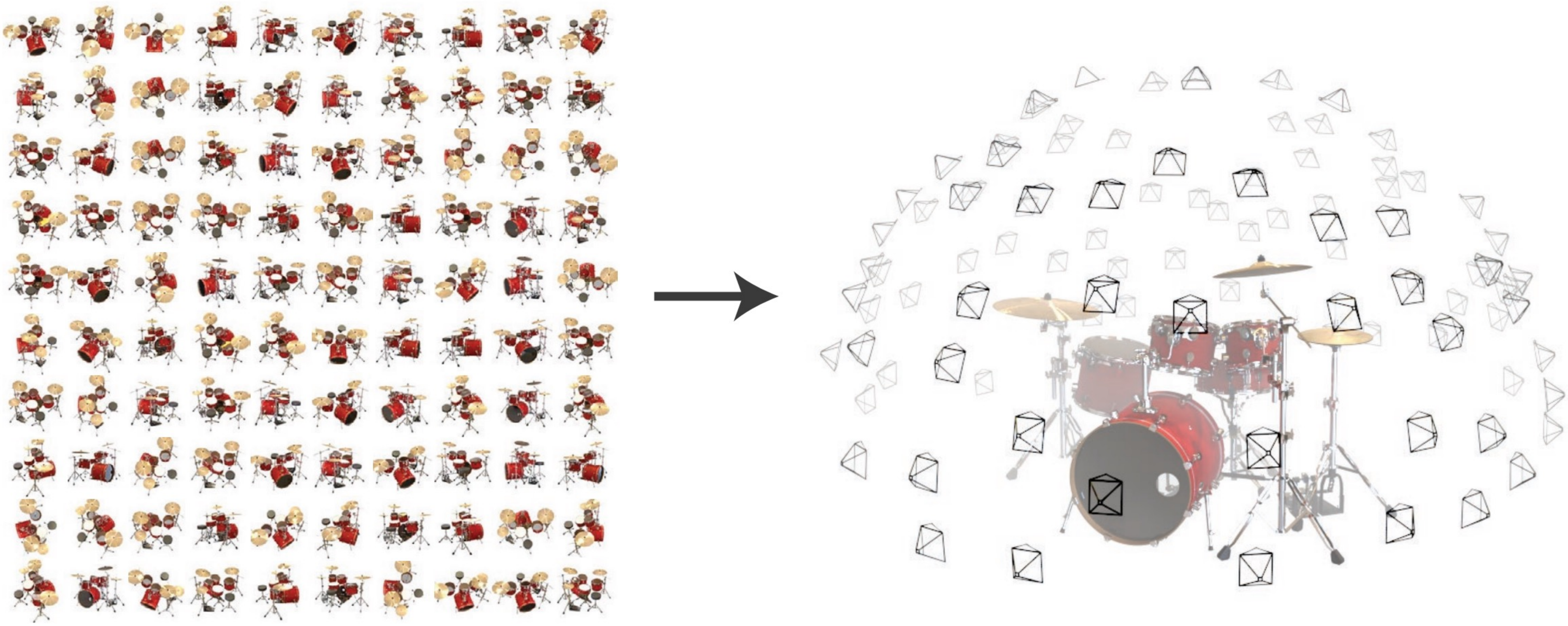
Spatio-Temporal Geometry Fusion for Multiple Hybrid Cameras using Moving Least Squares Surfaces, Eurographics 2014

Shape Acquisition

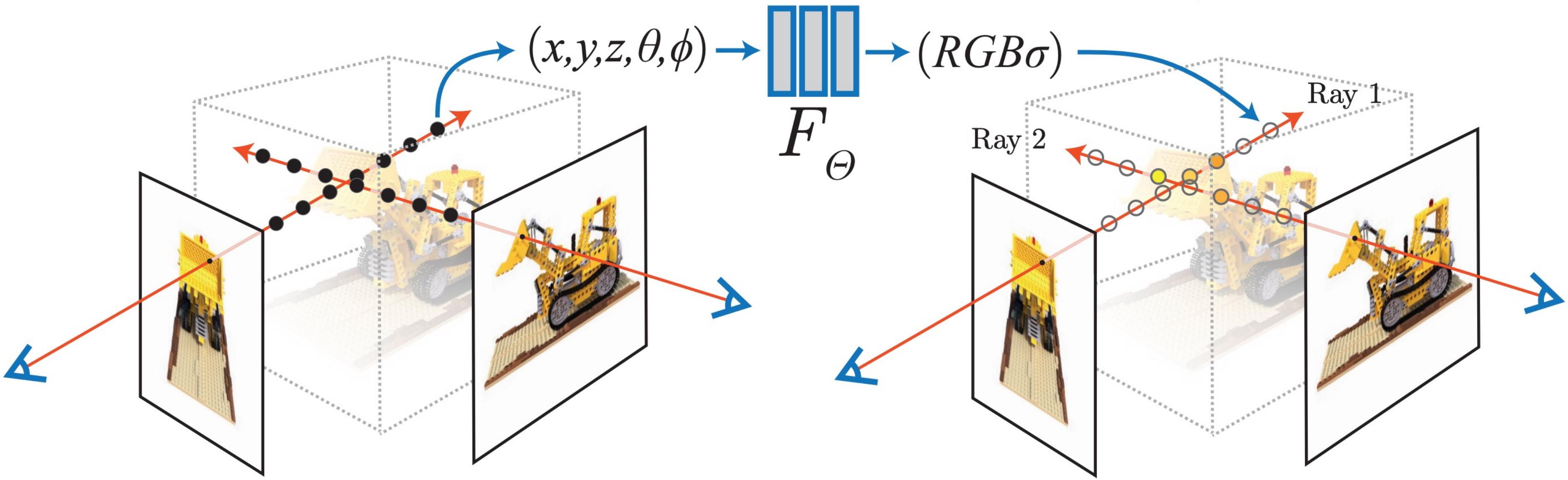
- Digitizing real world objects



Neural Radiance Fields



Neural Radiance Fields



Neural Radiance Fields

Neural Radiance Fields