Introduction to Probability

Session 13: Example Class

Mateja Jamnik, Thomas Sauerwald

University of Cambridge, Department of Computer Science and Technology
email: {mateja.jamnik,thomas.sauerwald}@cl.cam.ac.uk

Easter 2023

UNIVERSITY OF CAMBRIDGE

Plan for Today

3 worked out examples:

1. Application of Central Limit Theorem
2. Bias and MSE of Estimators
3. Local Maxima (“Best-so-far Candidates”) in the Secretary Problem

And plenty of time to answer your questions!

Example 1

Assume that an unknown fraction p of voters support a particular candidate. We poll $n = 100$ random voters and record by $\overline{X}_n := \frac{1}{n} \cdot (X_1 + X_2 + \cdots + X_n)$ the fraction of polled voters that support the candidate. Using the CLT, find an ϵ so that $\mathbf{P}\left[\left|\overline{X}_n - p\right| \leq \epsilon\right] \geq 0.95$.

Answer

Example 2 [source: Dekking et al., Exercise 20.3]

Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. samples from $Exp(\lambda)$. We would like to estimate the unknown mean $1/\lambda$. Let $T_1 := \overline{X}_n = \frac{1}{n} \cdot (X_1 + X_2 + \ldots + X_n)$ be the sample mean.

1. Define $M_n := \min(X_1, X_2, \ldots, X_n)$. What is the distribution of M_n?
2. Find an unbiased estimator T_2 for $1/\lambda$ based on M_n.
3. Which of the two estimators T_1 or T_2 is preferable?

Answer

Example 2 [source: Dekking et al., Exercise 20.3]

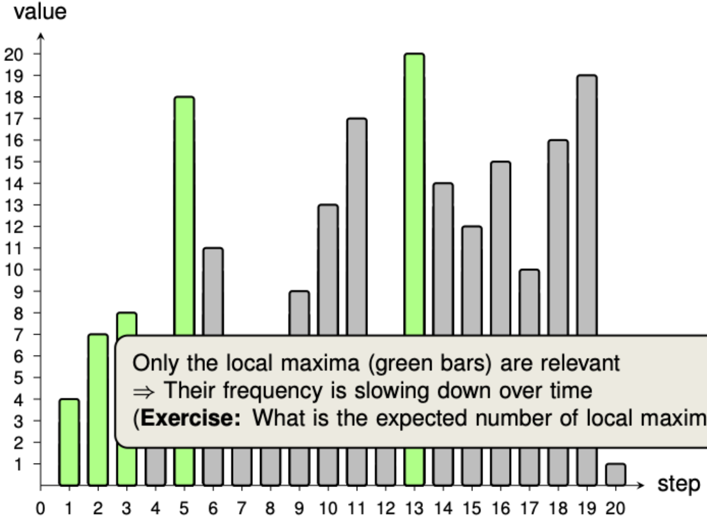
Suppose $X_1, X_2, \ldots, X_n$ are i.i.d. samples from $Exp(\lambda)$. We would like to estimate the unknown mean $1/\lambda$. Let $T_1 := \overline{X}_n = \frac{1}{n} \cdot (X_1 + X_2 + \ldots + X_n)$ be the sample mean.

1. Define $M_n := \min(X_1, X_2, \ldots, X_n)$. What is the distribution of M_n?
2. Find an unbiased estimator T_2 for $1/\lambda$ based on M_n.
3. Which of the two estimators T_1 or T_2 is preferable?

Answer

Reminder: Secretary Problem

unknown permutation:
4, 7, 8, 6, 18, 11, 3, 5, 9, 13, 17, 2, 20, 14, 12, 15, 10, 16, 19, 1.



Only the local maxima (green bars) are relevant
⇒ Their frequency is slowing down over time
(**Exercise:** What is the expected number of local maxima?)

Example 3

Consider the secretary problem, where the ranking of the n candidates is a random permutation. What is the expected number of “best-so-far” candidates?

Answer