Foundations of Computer Science:

Datatypes and Trees

Lecture 6

Anil Madhavapeddy

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Custom Types

Exceptions

Recursive Types

Custom Types

Custom Types

- So far, our types have been basic: `int`, `float` or `bool` types that are built into OCaml.

- In this lecture we introduce one of the coolest features of ML-style languages in the form of **custom datatypes**!

- We continue to improve the **abstraction** of our data away from the details of its **representation**.

Let’s describe a vehicle

```
# let number_of_wheels = function
      “bike” -> 2
    | “motorbike” -> 2
    | “car” -> 4
    | “lorry” -> 18
```

Let’s describe a vehicle

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```

```
# number_of_wheels “bike”
- : int = 2

# number_of_wheels “motorbke”
???
```

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```

How can we make illegal states unrepresentable?

An Enumeration Type

```
# type vehicle =
      Bike
    | Motorbike
    | Car
    | Lorry
```

An Enumeration Type

```
# type vehicle =
    Bike
  | Motorbike
  | Car
  | Lorry
```

- We have declared a new type `vehicle`
- Instead of representing any string, it can *only* contain the four constants defined.
- These four constants become the *constructors* of the `vehicle` type

An Enumeration Type

```
# type vehicle =
     Bike
   | Motorbike
   | Car
   | Lorry
```

- The *representation* in memory is more efficient than using strings.

- Adding new types of vehicles is straightforward by extending the definitions.

- Different custom types cannot be intermixed, unlike strings or integers.

Declaring functions on vehicles

```
# let wheels = function
  | Bike -> 2
  | Motorbike -> 2
  | Car -> 4
  | Lorry -> 18
val wheels : vehicle -> int = <fun>
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```
# let wheels = function
  | “bike” -> 2
  | “motorbike” -> 2
  | “car” -> 4
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val wheels : string -> int = <fun>
```

- The *representation* in memory is more efficient than using strings.
- Different custom types cannot be intermixed, unlike strings or integers.

Declaring functions on vehicles

```
# let wheels = function
  | Bike -> 2
  | Motorbike -> 2
  | Car -> 4
  | Lorry -> 18
val wheels : vehicle -> int = <fun>
```

```
# let wheels = function
  | Bike -> 2
  | Motorbike -> 2
  | Car -> 4
Warning 8: this pattern-matching is not exhaustive.
Here is an example of a case that is not matched:
Orange
val wheels : vehicle -> int = <fun>
```

- Adding new types of vehicles is straightforward by extending the definitions and fixing warnings.

Declaring functions on vehicles

```
# type vehicle = Bike
               | Motorbike of int
               | Car       of bool
               | Lorry     of int
```

- OCaml generalises the notion of enumeration types to allow *data* to be stored alongside each variant.

```
# Bike
# Motorbike 250
# Car true
# Lorry 500
```

Declaring functions on vehicles

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               | Car       of bool
               | Lorry     of int
```

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```
# type vehicle = Bike
               | Motorbike of int  (* engine size in CCs *)
               | Car       of bool (* true if a Reliant Robin *)
               | Lorry     of int  (* number of wheels *)
```

Declaring functions on vehicles

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An OCaml comment allows annotation of source code

Declaring functions on vehicles

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# type vehicle = Bike
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- OCaml generalises the notion of enumeration types to allow *data* to be stored alongside each variant.

- Even though they have different data, they are all of type `vehicle` when wrapped by the constructor.

```
# [ Bike; Car true; Motorbike 450 ]
- : vehicle list
```

A finer wheel computation

```
# let wheels = function
  | Bike -> 2
  | Motorbike _ -> 2
  | Car robin -> if robin then 3 else 4
  | Lorry w -> w
```

- A Bike has two wheels.
- A Motorbike has two wheels.
- A Reliant Robin has three wheels; all other cars have four.
- A Lorry has the number of wheels stored with its constructor.

A finer wheel computation

```
# let is_reliant_robin = function
    | Car true -> true
    | _ -> false
```

Exceptions

Exceptions

- During a computation, what if **something goes wrong**?
  - Division by zero
  - Pattern matching failure
- **Exception handling** allows us to recover from these:
  - **Raising** an exception abandons the current expression
  - **Handling** the exception attempts an alternative
- Raising and handling can be separated in the source code

Exceptions

```
# exception Failure
exception Failure

# exception NoChange of int
exception NoChange of int

# raise Failure
Exception: Failure.
```

- Each `exception` declaration introduces a distinct type of exception that can be handled separately.

- Exceptions are like enumerations and can have data attached to them.

Exceptions

```
# try
    print_endline "pre exception";
    raise (NoChange 1);
    print_endline "post exception";
  with
    | NoChange _ ->
        print_endline "handled a NoChange exception"
Line 3, characters 5-23:
Warning 21: this statement never returns (or has an unsound type.)
pre exception
handled a NoChange exception
- : unit = ()
```

- `raise` dynamically jumps to the nearest `try/with` handler that matches that exception

- Unlike some languages, OCaml does not mark a function to indicate that an exception might be raised.

Exceptions

Install exception handler for enclosing block

```
# try
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    raise (NoChange 1);
    print_endline "post exception";
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Change : a recap

```
let rec change till amt =
  if amt = 0 then
    [ [] ]
  else
    match till with
    | [] -> []
    | c::till ->
        if amt < c then
          change till amt
        else
          let rec allc = function
          | [] -> []
          | cs :: css -> (c::cs) :: allc css
          in
            allc (change (c::till) (amt - c)) @
                 change till amt
```

Change with *backtracking*

```
# exception Change
  let rec change till amt =
    if amt = 0 then
      []
    else
      match till with
      | [] ->
          raise Change
      | c::till ->
          if amt < 0 then
            raise Change
          else
            try
               c :: change (c::till) (amt - c)
            with Change ->
               change till amt
exception Change
val change : int list -> int -> int list = <fun>
```

Change with *backtracking*

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              change till amt
exception Change
val change : int list -> int -> int list = <fun>
```

Attempt the solution

Remove some change and retry if stuck

Change wit

```
# exception Change
  let rec change till amt =
    if amt = 0 then
      []
    else
      match till with
      | [] ->
          raise Change
      | c::till ->
          if amt < 0 then
            raise Change
          else
            try
               c :: change (c::till) (amt - c)
            with Change ->
               change till amt
exception Change
val change : int list -> int -> int list = <fun>
```

```
change [5; 2] 6
  ⇒ 5::change [5; 2] 1 with C -> change [2] 6
  ⇒ 5::(5::change [5; 2] -4) with C -> change [2] 1
                             with C -> change [2] 6
  ⇒ 5::(change [2] 1) with C -> change [2] 6
  ⇒ 5::(2::change [2] -1) with Chang -> change [] 1
                          with C -> change [2] 6
  ⇒ 5::(change [] 1) with C -> change [2] 6
  ⇒ change [2] 6
  ⇒ 2::(change [2] 4) with C -> change [] 6
  ⇒ 2::(2::change [2] 2) with C -> change [] 4
                         with C -> change [] 6
  ⇒ 2::(2::(2::change [2] 0)) with C -> change [] 2
                              with C -> change [] 4
                              with C -> change [] 6
  ⇒ 2::(2::[2]) with C -> change [] 4
                with C -> change [] 6
  ⇒ 2::[2; 2] with C -> change [] 6
  ⇒ [2; 2; 2]
```

Recursive Types

Binary Trees

```
# type 'a tree =
    Lf
  | Br of 'a * 'a tree * 'a tree
```

Binary Trees

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```

- A data structure with multiple branching is called a **tree**.
- Trees are nearly as fundamental a structure as lists.
- Each node is either a **leaf** (empty) or a **branch** with a label and two subtrees.

Binary Trees

```
# type 'a tree =
    Lf
  | Br of 'a * 'a tree * 'a tree
```

“Polymorphic” type

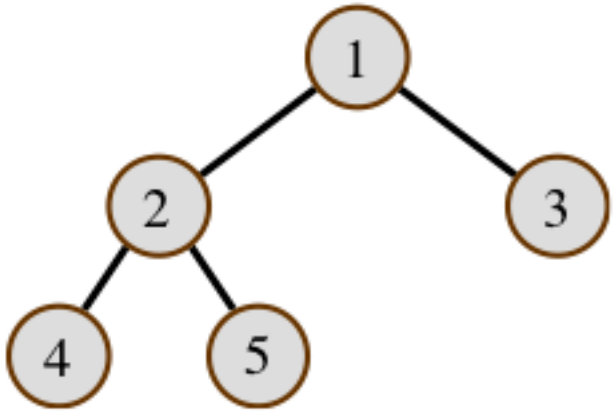
Binary Trees

“Polymorphic” type

```
# type 'a tree =
    Lf
  | Br of 'a * 'a tree * 'a tree
```

`int tree`

```
# Br(1, Br(2, Br(4, Lf, Lf),
            Br(5, Lf, Lf)),
      Br(3, Lf, Lf))
```



Binary Trees & Lists

```
# type 'a tree =
    Lf
  | Br of 'a * 'a tree * 'a tree
```

```
# type 'a mylist =
| Nil
| Cons of 'a * 'a mylist

# Cons (1, Cons (2, Cons (3, Nil)))
- : int mylist
```

Polymorphism & Recursion

```
# type 'a tree =
    Lf
  | Br of 'a * 'a tree * 'a tree
```

Polymorphic and Recursive

```
type shape =
| Null
| Join of shape * shape
```

Recursive

```
type 'a option =
| None
| Some of 'a
```

Polymorphic

Simple Operations on Trees

```
(* number of branch nodes *)
# let rec count = function
  | Lf -> 0
  | Br (v, t1, t2) -> 1 + count t1 + count t2
val count : 'a tree -> int = <fun>

(* length of longest path *)
# let rec depth = function
  | Lf -> 0
  | Br (v, t1, t2) -> 1 + max (depth t1) (depth t2)
val depth : 'a tree -> int = <fun>
```

- Use pattern matching to build expressions over trees
- The invariant $\text{count}(t) \leq 2^{\text{depth}(t)} - 1$ holds above