1999 Paper 1 Question 5

Foundations of Computer Science

This question has been translated from Standard ML to OCaml

"We face the Year 2000 crisis because programmers did not apply the principles of data abstraction." Discuss. [4 marks]

Your employer asks you to implement a dictionary. The pattern of usage will consist of taking an empty dictionary and performing many insertions and lookups. You must choose one of three data structures. Each requires $O(\log n)$ time for the lookup and update operations, where n is the number of items in the dictionary. They are (1) binary search trees, which take $O(\log n)$ time in the average case; (2) balanced trees, which need complicated algorithms but take $O(\log n)$ time in the worst case; (3) self-adjusting trees, which take $O(\log n)$ amortised time in the worst case.

Explain the differences between the three notions of $O(\log n)$ time. Argue that any of the three data structures might turn out to be the best, depending upon further details of the application. If no further details are available, which of the three is the safest choice? [8 marks]

An algorithm requires $T(n)$ units of space given an input of size n, where the function T satisfies the recurrence

$$T(1) = 1$$
$$T(n) = T(n/2) + n \qquad (n > 1).$$

Express the algorithm's space requirement using O-notation, carefully justifying your answer. [8 marks]

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