Quantum Complexity Theory

The Quantum Cook-Levin theorem



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Quote of the day

"Information is physical"

(Rolf) Landauer's principle



Quantum Cook-Levin Theorem

The 5-local Hamiltonian problem is QMA-complete

Is it in NP?

Why is it not trivially in QMA?

Proof sketch: Local Hamiltonians are in QMA

Merlin sends a candidate ground state ρ

Shift and rescale terms and thresholds so $0 \leq H_i \leq I$

Arthur picks a uniformly random local term

Measure the POVM $\{H_i, I - H_i\}$. Reject on the first outcome

Local Hamiltonians are in QMA

For $H = \sum_{i=1}^{m} H_i$, rejection is the average energy:

$$\Pr[\text{reject}] = \frac{1}{m} \sum_{i=1}^{m} \mathrm{Tr}(H_i \rho) = \frac{\mathrm{Tr}(H\rho)}{m}$$

YES: a ground state has energy at most α

$$\Pr[\text{accept}] \geq 1 - \frac{\alpha}{m}$$

NO: every state has energy at least β

$$\Pr[\text{accept}] \leq 1 - \frac{\beta}{m}$$

The acceptance gap $(\beta - \alpha)/m \geq 1/\mathrm{poly}(n)$ can be amplified (M-W)

Quantum Mythology



Did Kitaev prove the QMA-completeness of the Local Hamiltonian problem while on a flight to Chicago?

Asked 2 years, 2 months ago Modified 2 years, 1 month ago Viewed 522 times

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I believe the first publication of the proof of the QMA-completeness of the Local Hamiltonian Problem is as portions of a chapter in the book *Classical and Quantum Computation*, by Kitaev, Shen, and Vyalyi, published in Russian in 1999 (with an English translation later in 2003). Up until the book publication, the research papers that referenced the QMA-completeness of the LHP had cited a talk by Kitaev and titled "Quantum NP", at the second QIP/AQIP conference hosted by DePaul University in Chicago in January 1999. The agenda for the conference is still available on the web, and looking at the presenters and the abstracts of their presentation gives me shivers.

I have heard a rumor that Kitaev had worked out the details of the problem and the proof of its QMA-completeness while flying to Chicago. The story I had heard was that the conference encouraged presenters to discuss something novel, and Kitaev started thinking about the Cook-Levin theorem and wondering how to port it over to quantum computing. If so, to me it's a pretty fascinating anecdote, that such a foundational theorem was cobbled together on a flight in preparation for a talk about quantum computing, only four and a half years after Shor '94. I can imagine a pre-9/11, four-hour airplane flight from the west coast of the US to O'Hare Airport, with a handsome young Russian gentleman scribbling away on theoretical computer science and the limits it can teach us about quantum mechanics and the physical world itself.

The Overflow

- A green exit … the work hep…
- Taking a look…

Featured on M…

- We've rework… deletion. Let's…
- How can we i… handling?

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Kitaev did not prove the QMA-completeness of LHP on a flight to Chicago in early 1999. He did so a couple of days before he flew from Russia to Israel in 1998.

In particular, around then D. Aharanov had heard of the works of Kitaev, then in Russia, and suggested he travel and meet with her and her advisor A. Wigderson in Israel (at Hebrew University.) Wigderson agreed and Aharanov extended the invitation to Kitaev. Kitaev, being a physicist, wanted to be able to converse about theoretical computer science with Wigderson and Aharanov, and this is when he began to port Cook-Levin over to quantum computing - -two days before his flight.

Aharanov explains this better than I ever could, at least a couple of times in some of her lectures about quantum computing and the limits it can teach us about physical reality. See, here at about the 36-minute mark, and also here at about the 27-minute mark.

The Feynman-Kitaev Hamiltonian

Local penalties enforce the same three conditions:

Input check

The input is x and the ancillas are zero

Propagation check

Consecutive snapshots differ by the next gate

Output check

The final output qubit accepts

$$H = H_{in} + H_{prop} + H_{out}$$

QMA-hardness: the challenges

What goes wrong when we quantise the classical Cook-Levin theorem?

Problem 1: Honest computation need not accept with certainty

Idea: Soundness amplification (also for MA)

Problem 2: Local consistency does not imply global consistency

Idea: Coherent history state

Problem 3: The clock itself threatens locality

Idea: Unary clock

Problem 4: Non-discrete violations

Idea: Quantitative spectral argument

First step: amplification

Fix an instance x of any problem in QMA

Amplify the verifier so its error is exponentially small

$$\varepsilon = 2^{-\mathrm{poly}(n)}, \quad V = U_T \cdots U_2 U_1, \quad T = \mathrm{poly}(n)$$

YES: some proof is accepted with probability $\geq 1 - \varepsilon$

NO: every proof is accepted with probability $\leq \varepsilon$

Key idea: history state

A quantum version of the Cook-Levin tableau

$$|\Omega\rangle = \frac{1}{\sqrt{T+1}} \sum_{t=0}^{T} |t\rangle_C \otimes |\Omega_t\rangle$$

$$|\Omega_t\rangle = U_t \cdots U_1 (|x\rangle_X \otimes |\psi\rangle_P \otimes |0^a\rangle_A)$$

The clock labels each snapshot of the computation

All snapshots appear in coherent superposition

The honest proof is a low-energy history state