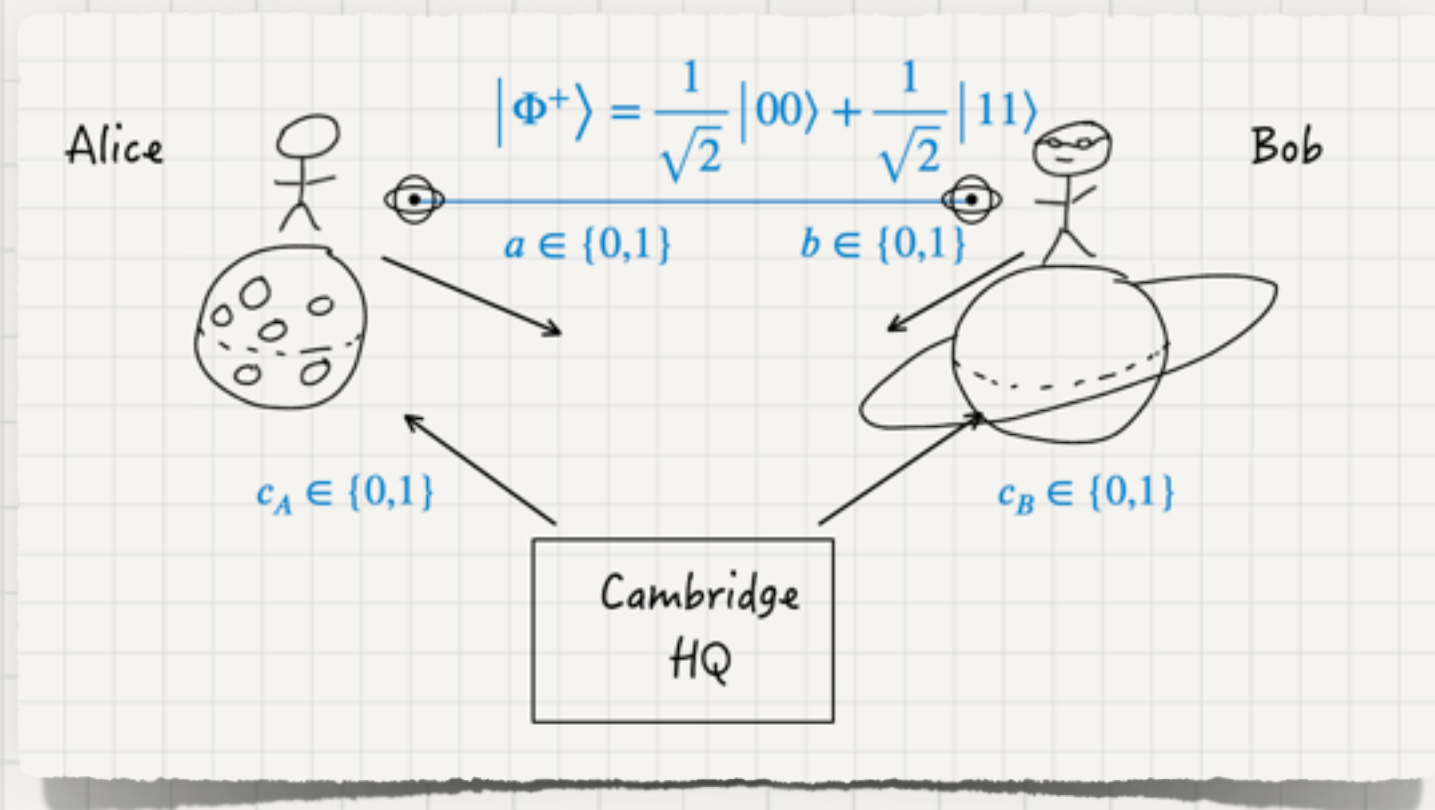


Quantum Complexity Theory

Lecture 2: Bell's inequality and MIP*

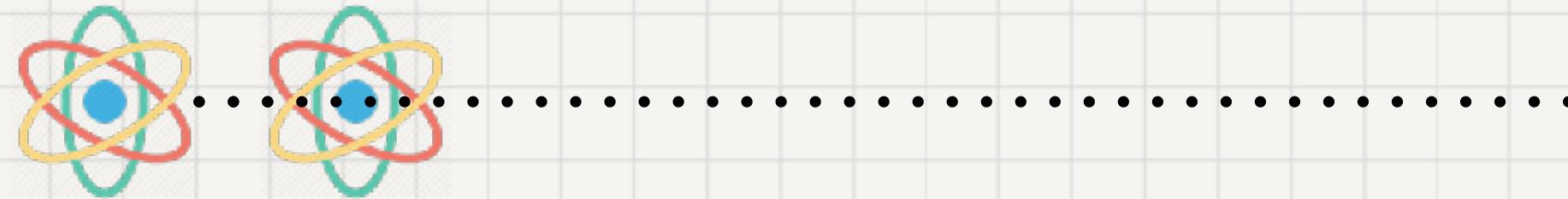


Tom Gur



UNIVERSITY OF
CAMBRIDGE

Experiment



$\sim 132 \text{ km}$

$1.4 * 10^{-15}$ light years

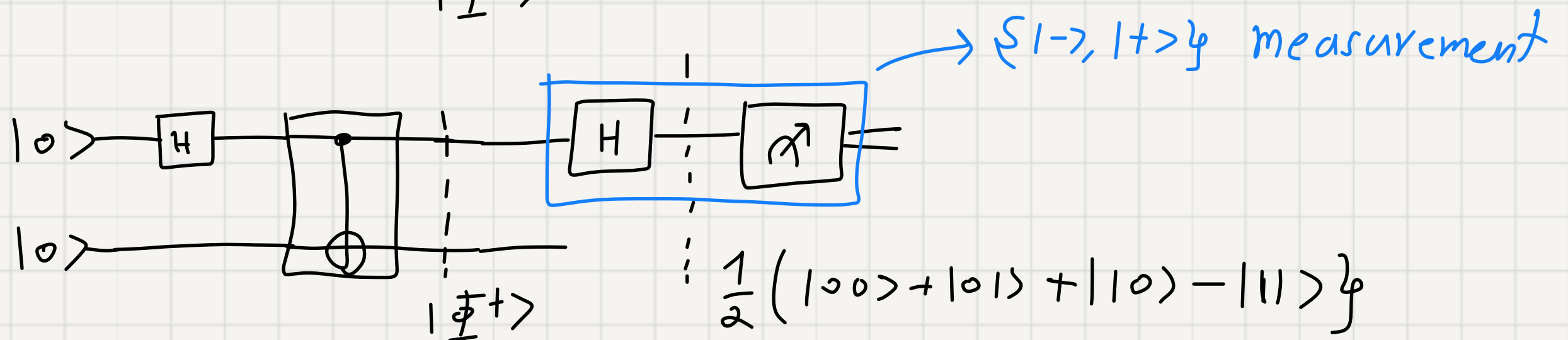
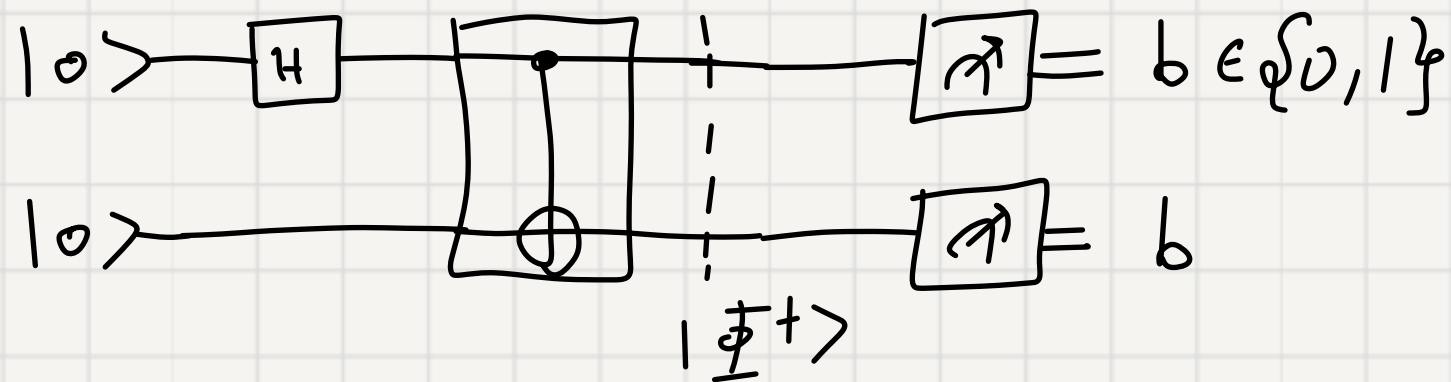
$\sim 0.4 \text{ ms}$ for light to travel



change one, and the other would be immediately effected!

Einstein, Podolsky, Rosen 1935

FTL?!
!!



What is Bob's state?

- If Alice measured $|0\rangle$: $|0\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |0+\rangle$
- If Alice measured $|1\rangle$: $|1\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |1-\rangle$

Quote of the day

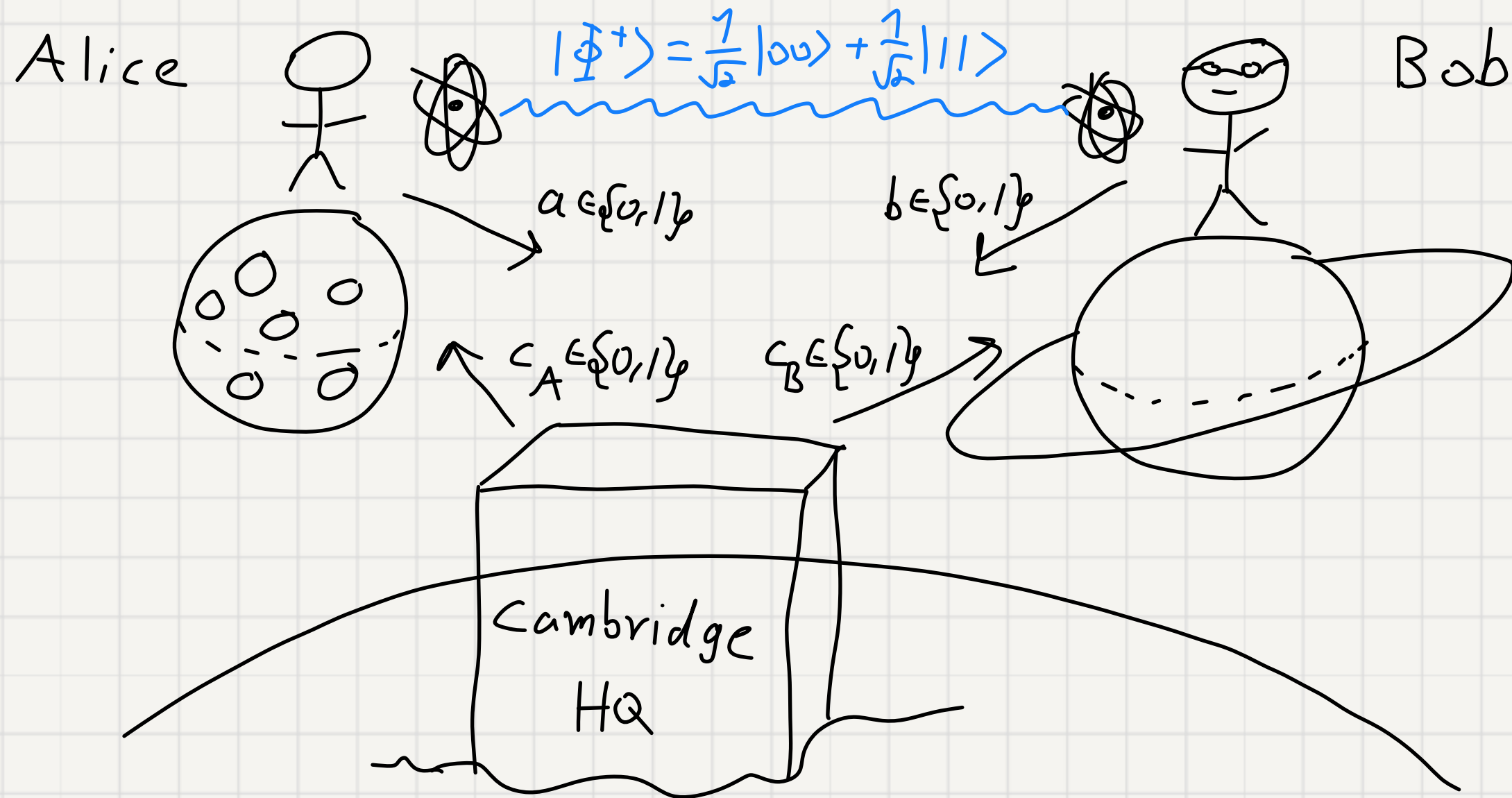
1947



I cannot seriously believe in quantum mechanics, because the theory cannot be reconciled with the idea that physics should represent a reality in time and space, free from spooky actions at a distance."

— Albert Einstein

Clauser, Horne, Shimony, Holt 1969 (CHSH game)



c_A, c_B chosen at random. Goal: If $c_A = c_B = 1$ $a \neq b$

o/w

$a = b$

Def A Bell inequality of game G is an upper bound on the max success prob. of a classical strategy

Claim $\text{Bell}(\langle HSH \rangle) < 0.75$

Proof By convexity, w.l.o.g., the strategy is deterministic.

Observe the game is won iff

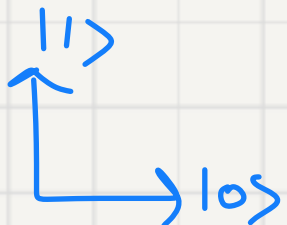
$$a(\langle A \rangle) \oplus b(\langle B \rangle) = \langle A \rangle \cdot \langle B \rangle$$

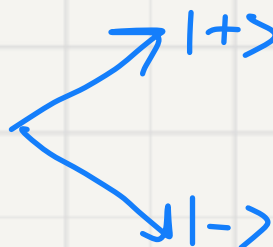
can only hold for $3/4$ vals of $\langle A \rangle, \langle B \rangle$ $\square$

Thm $\exists$ quantum strategy that wins CHSH w.p. ≥ 0.85

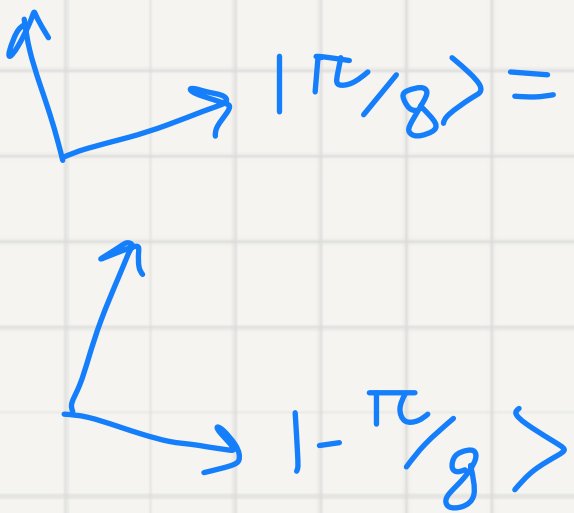
This was verified experimentally!

Proof consider the strategy:

Alice: If $C_A = 0$, measure 

If $C_A = 1$, measure 

Send $a=0$ if $|0\rangle/|1+\rangle$
 $a=1$ o/w

Bob: If $C_B = 0$, measure  $| \pi/8 \rangle = \cos \frac{\pi}{8} |0\rangle + \sin \frac{\pi}{8} |1\rangle$

If $C_B = 1$, measure

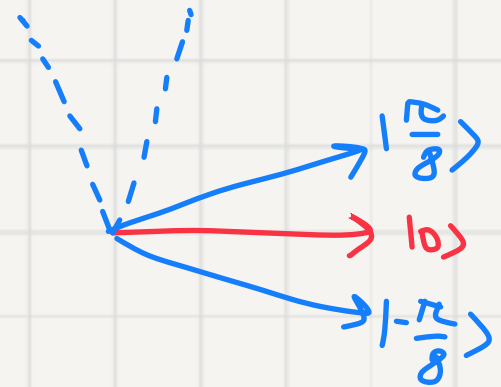
Send $b=0$ if $| \pm \frac{\pi}{8} \rangle$
 $b=1$ o/w

Analysis

Sps $c_A = 0$ and Alice measures $|0\rangle$. Then $a=0$.

Bob's qubit collapsed to $|0\rangle$. Win iff $b=a=0$.

$$\Pr\left[\left|\frac{\pi}{8}\right\rangle\right] = \Pr\left[\left|-\frac{\pi}{8}\right\rangle\right] = \cos^2\frac{\pi}{8} \geq 0.85$$



Same unless $c_A = c_B = 1$ (check !)

Then Alice measures in $\{|+\rangle, |-\rangle\}$. Sps $|-\rangle$. $a=1$

Bob has $|-\rangle$. Win iff $b=0$.

$$\Pr\left[\left|-\frac{\pi}{8}\right\rangle\right] = \cos^2\frac{\pi}{8} \geq 0.85$$

$\Rightarrow$ local realism is false

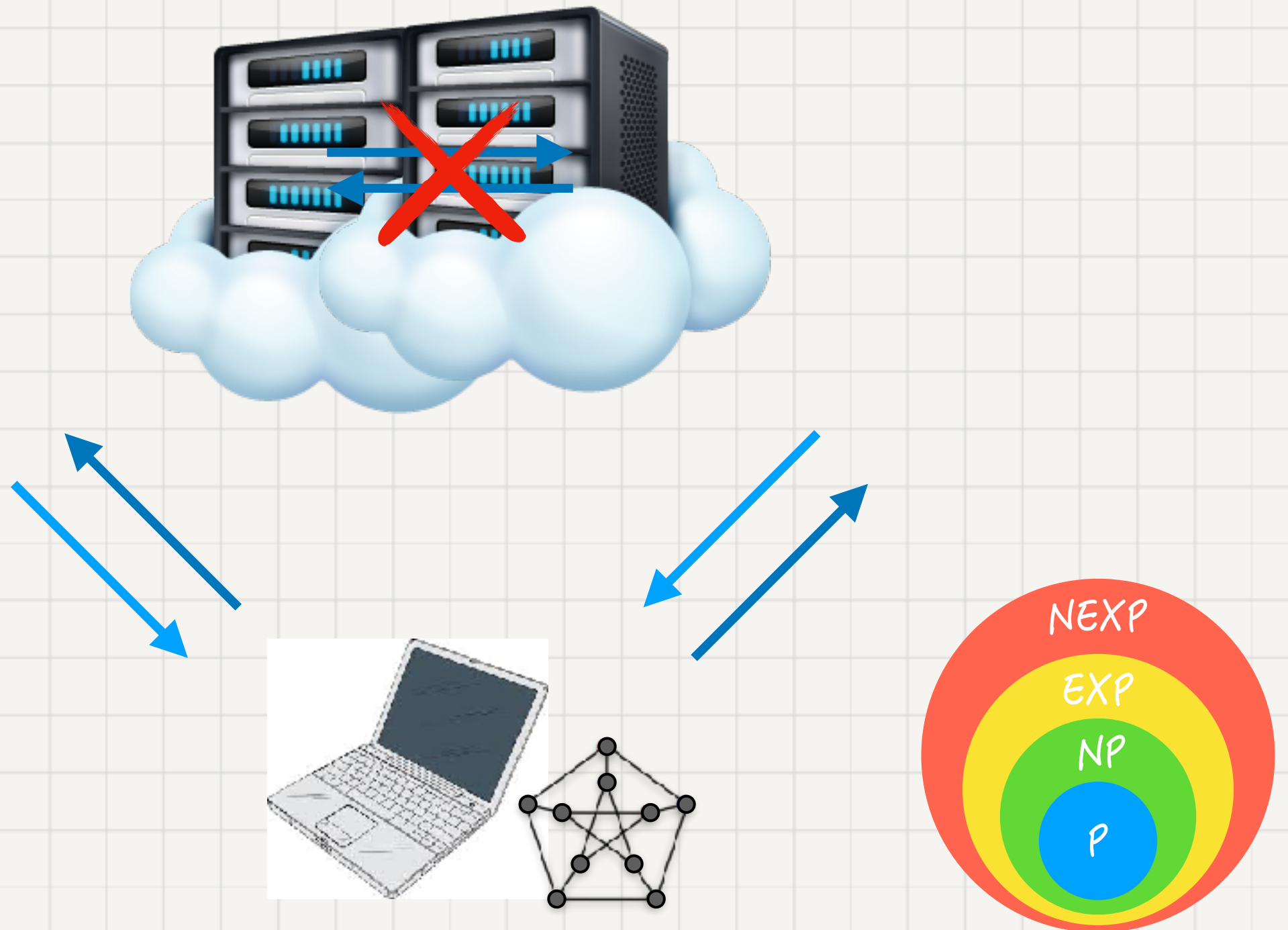


Discussion

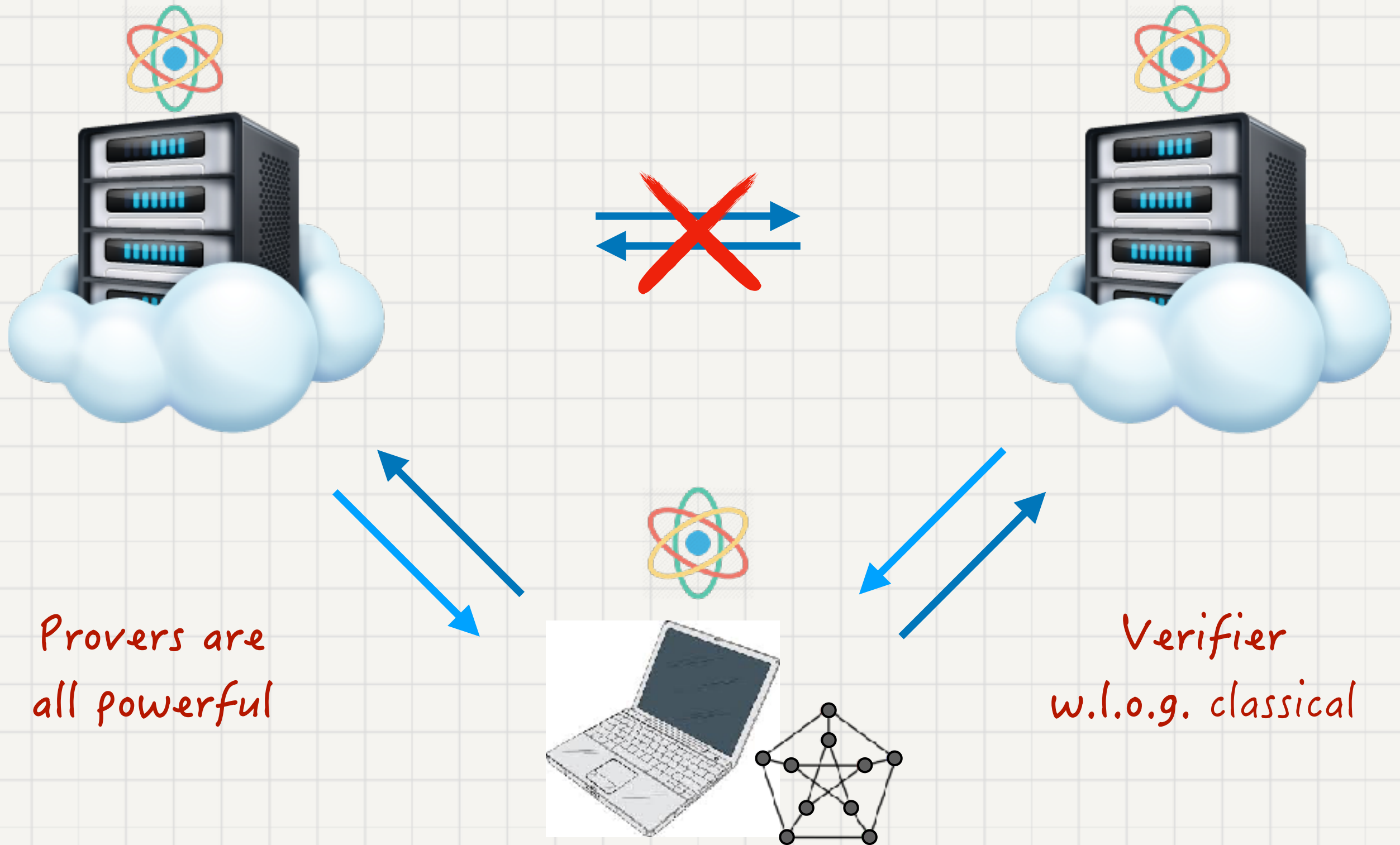
- Magic square game
- Tsierlson bounds
- Hidden local variable vs FTL
- If we could clone, we could learn the qubit...
- Mixed states

Rigidity!

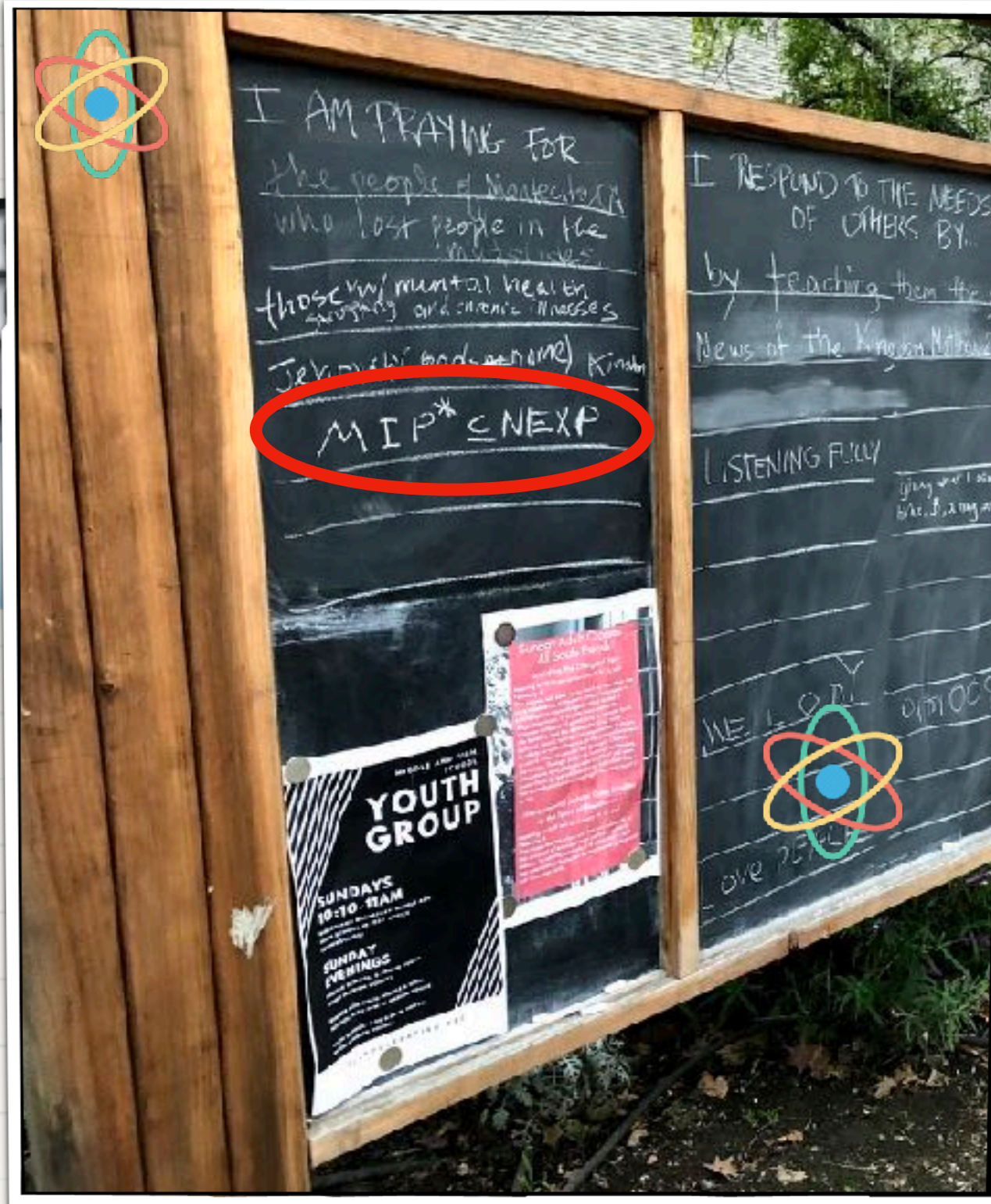
CS Perspective: Multi-prover Interactive Proofs (MIP)



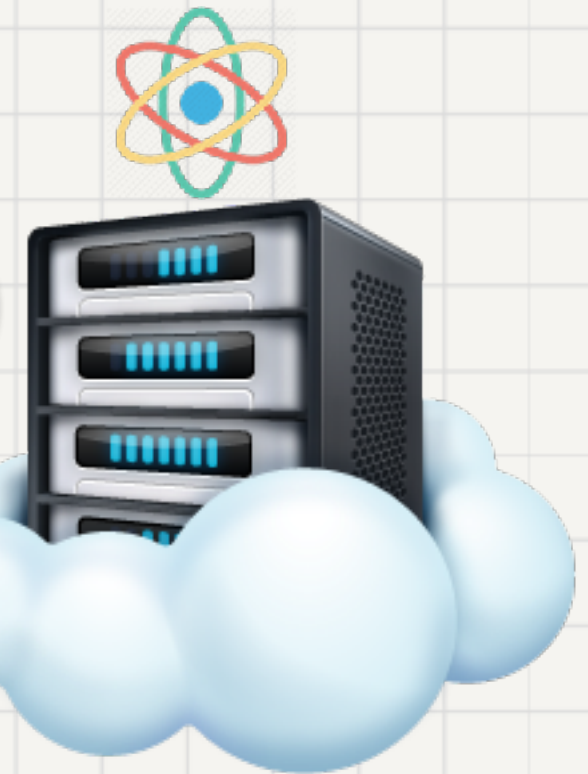
Why does this matter? ← Quantum MIP



Quantum MIP



$|\psi\rangle$



$NEXP \subseteq MIP^*$

[Ito-Vidick 12]

Upper bound?

$$\underline{MIP^* = RE}$$

