Notes

Pauli Group is generated by $\{I, X, Y, Z\}$

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$; $Y = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

→ Square them to get identity

✡ $XZ = iY$

Clifford Group is generated by $\{CX, S, H\}$

$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$ $H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$CX = \begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 0 & 1 \\ & & 1 & 0 \end{bmatrix}$$

Pauli group is subgroup of Clifford group. $[P \leq C]$





Gifford group is classically simulable but not universal.

Table am form

CX gate ↔



and

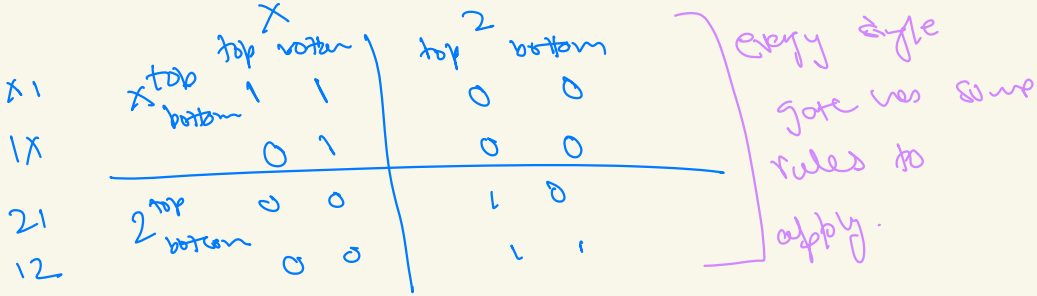


and





- global phase are not seen & squared gives identity.



Gottesman-Knill theorem

⇒ you can classically efficiently simulate any Clifford circuits.

⇒ But Clifford circuits are not universal for computation

Simple universal gate set is $\{X, Z(\frac{\pi}{4}), CX\}$

→ can exactly synthesise any 2^n by 2^n unitary matrix.

Quantum Error Correction (QEC)

idea → different gates are fault-tolerant in different codes.

Eastin-Knill theorem: For any nontrivial QEC code that exactly detects every single subsystem error cannot have a universal set of transversal logic unitary gates

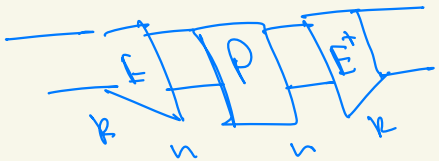
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no-go theorem.

Definitions :-

1. logical : in a QEC, you physically implement some computation P & decode to get logical computation L.

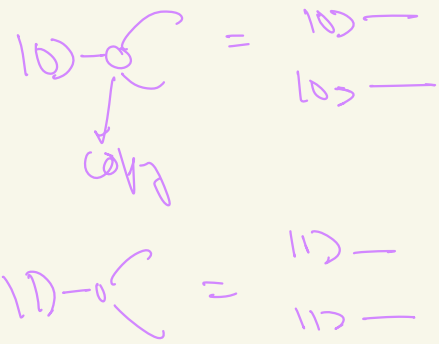
→ the encoding map E relate P to L.

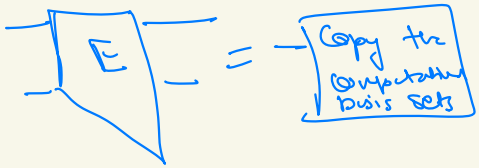


→ E must be an isometry.

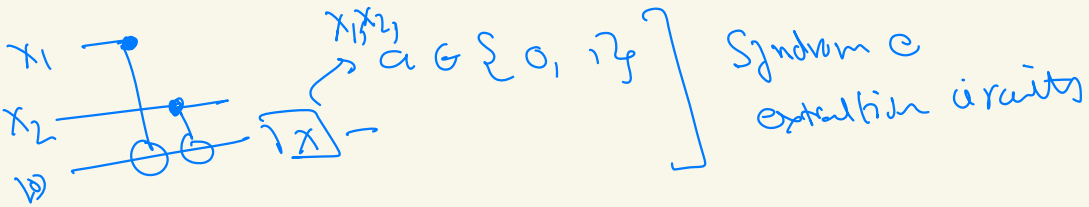
$E; E^+ = I_k$ & $E^+; E \neq I_n$

2 bit repetition code :-

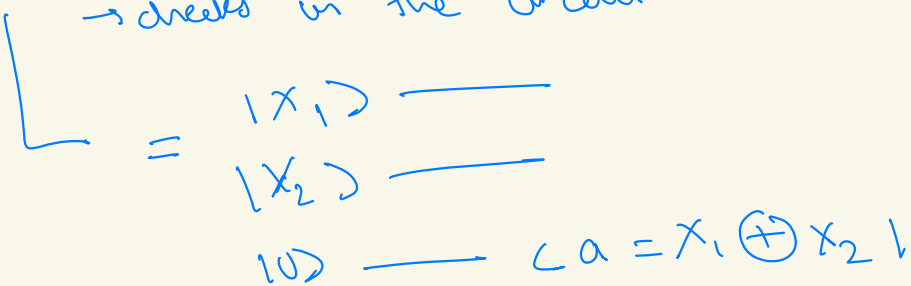




→ Prop. of a bit flip error → X error



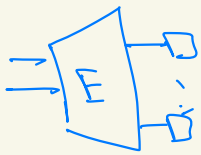
→ checks in the circuit



→ measures whether you have left the codespace without hurting the logical information.

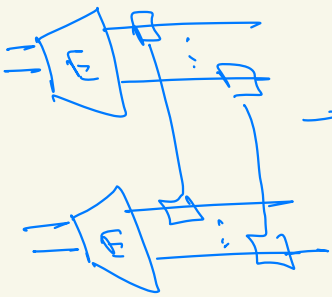
2. Transversal :- non-entangling between physical qubits in the same code block of n qubits.

Example (a)



→ implements on n physical qubits

(b)



→ b/w code blocks but it doesn't allow error propagation.

Punchline: Simulating cliffords easy: just gates is also easy. Clifford + T set is approximately universal in quantum computation.