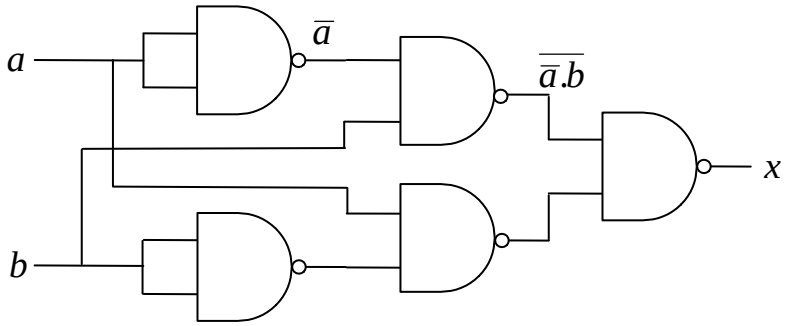
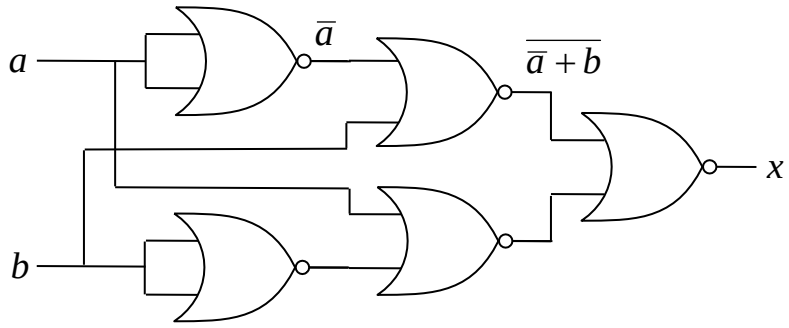
IA – Digital Electronics

Examples Paper 1 – Combinational Logic

1. Complete the truth tables for the following digital logic circuits.



a	b	$\overline{a}$	$\overline{\overline{a}.b}$			x
0	0					
0	1					
1	0					
1	1					

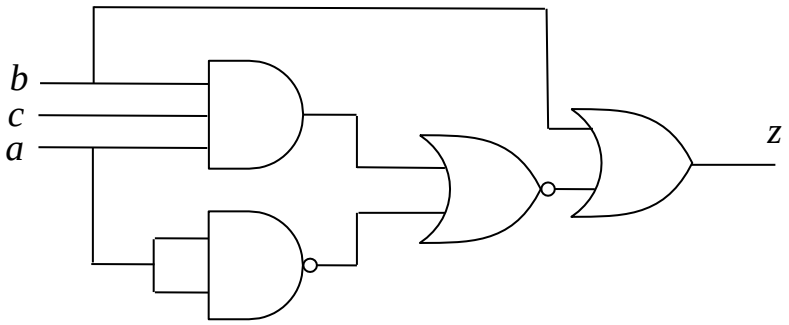


a	b	$\overline{a}$	$\overline{\overline{a}+b}$			x
0	0					
0	1					
1	0					
1	1					

2. Use Boolean algebra to prove the following identities:

$$a.b.c + a.b.\overline{c} = a.b$$
$$a.(\overline{a} + b) = a.b$$
$$a.b + \overline{a}.c = (a + c).(\overline{a} + b)$$
$$(a + c).(a + d).(b + c).(b + d) = a.b + c.d$$

3. The following circuit does not make efficient use of logic gates. Write a Boolean expression for *z*, and hence show how *z* can be realised more efficiently.



4. A logic 'voter' circuit has 4 inputs *a*, *b*, *c*, *d* and one output *v*. The output is to be logic 1 if any 3 or all 4 inputs are at logic 1. Design a circuit using AND and OR gates to satisfy this requirement.

5. Devise circuits to solve question 4 if

(a) NAND gates only;
(b) NOR gates only are to be used.

6. Using a Karnaugh map, write simplified sum of products expressions for f and $\bar{f}$ where,

$$f = \bar{a}.\bar{d} + \bar{b}.\bar{c} + \bar{a}.b.\bar{c}.d$$

and $a.\bar{b}.c.\bar{d}$ is a don't care state.

7. A three variable function is given by:

$$f = a.\bar{b}.\bar{c} + a.\bar{b}.c + a.b.c$$

Find the simplest sum of products form for f using a Karnaugh map. Express *f* using:

(a) NAND gates only;
(b) NOR gates only. (Hint: try mapping $\bar{f}$ for fewest gates)

8. The function f in question 7 can be written in the form:

$$f = 100 + 101 + 111 = \sum(4,5,7)$$

Find the simplified sum of products and product of sums forms for the four variable function *g* where:

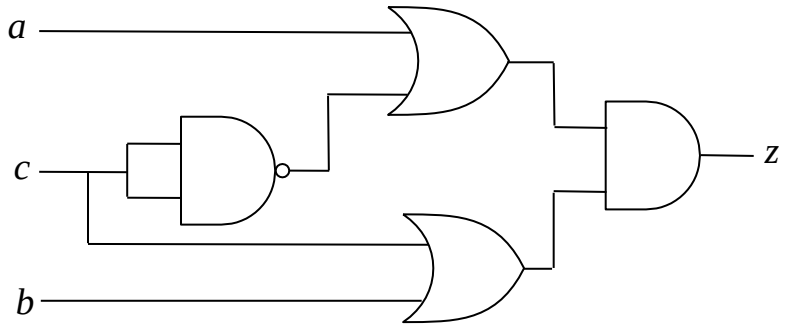
$$g = \sum(5,6,7,8)$$

and terms 10 to 15 inclusive are don't care states. Take *abcd* to be the four variables, with *a* the most significant.

9. The months of the year are coded in binary with January represented by $A_3, A_2, A_1, A_0 = (0001)$ and December by (1100). Find a simplified sum of products expression in terms of A_3, A_2, A_1, A_0 for the months without an r in their name.

Show that a simpler expression is obtained by changing the coding so January is represented by (0000) and December by (1011).

10. Each gate in the following circuit has a propagation delay of τ seconds.



(a) Draw a timing diagram showing the output of each gate for $a = b = 0$; and c initially 0, switching to 1 for a time t ($t >> \tau$), and then returning to 0 . Hence show that a static hazard exists. Is it a static 1 or static 0 hazard?

(b) Write down a product of sums expression for z from the circuit and use de Morgan's theorem to obtain a sum of products expression for $\bar{z}$.

(c) Draw a Karnaugh map for $\bar{z}$ and thus show how the hazard can be removed by adding one more OR gate to the circuit.

11. A 4 variable expression g has minterms $\sum(0,1,2,5,6,7,8,9,10,14)$. Take *ABCD* to be the four input variables, with *A* the most significant bit. Use the Quine-McCluskey (Q-M) method to yield a simplified expression in the sum of products (SOP) form.

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Relevant IA Paper 2 Tripos questions include:

Q1-2023, Q1-2022, Q1-2021, Q1-2020, Q1-2019, Q1-2018.

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