

Taming functions

□

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Translations

Functions are hard to handle

Functions



Translations

Reading

```
let map = fun f → fun l →  
  let rec loop = fun l →  
    match l with  
    | [] → []  
    | x :: xs → f x :: loop xs  
  in loop l
```

nesting makes program
transformation difficult

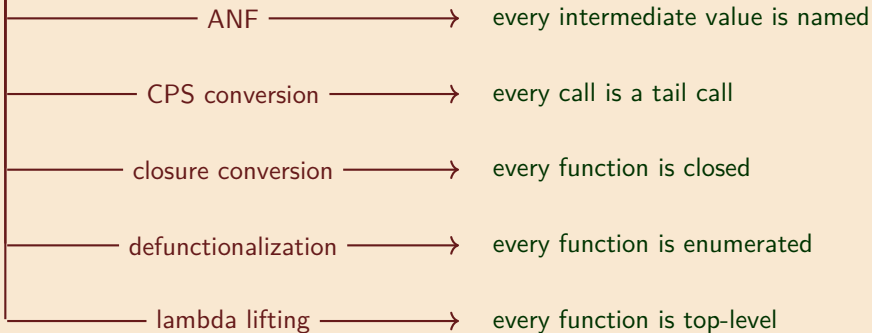
can't know all possible
functions in advance

analysis is difficult

control flow is implicit/unclear

We'll consider *total* translations that introduce syntactic invariants:

$\lambda x.e$



How should we view these translations?

Functions



Translations are *not optimizations*

Translations don't make programs faster (and may well make them slower)

Translations don't use heuristics, and aren't sensitive to structure

Translations

Translations are *representation changes*

The translation targets are *inconvenient as programming languages*

Changing representation makes analysis and further translation easier

Reading

Translations by example

Translation to A-normal form

Functions

```
let map = fun f → fun l →  
  let rec loop = fun l →  
    match l with  
    | [] → []  
    | x :: xs →  
      let y = f x in  
      let ys = loop xs in  
      let v = y :: ys in  
      v  
  in let v = loop l in v
```

Intermediate expressions *named*

Arguments always *trivial*
(values or variables)

Translations

Reading

Full details: *The Essence of Compiling with Continuations* (PLDI 1993)
Cormac Flanagan, Amr Sabry, Bruce F. Duba, Matthias Felleisen

Translation to continuing-passing style

Functions

Computed values passed to continuations,
not returned

Every function passed a *continuation k*,
the *rest of the computation*

```
let map_cps = fun f k → k (fun l k →  
  let rec loop = fun l k →  
    match l with  
    | [] → k []  
    | x :: xs → f x (fun y →  
      loop xs (fun ys →  
        k (y :: ys)))  
  in loop l k)  
  
let id x = x  
let map = fun f l → map_cps (fun x k → k (f x)) id l id
```

Every call a tail call

Translations

Reading

Closure conversion

Functions

All functions closed (i.e. C-style function pointers)

Free-variable environments bundled with functions

```
type ('a, 'b) clo =  
  Clo : ('a → 'env → 'b) * 'env → ('a, 'b) clo
```

```
let app (Clo (f, env)) x = f x env
```

```
let fp3 = fun l env → match l with  
  | [] → []  
  | x :: xs → app env.f x :: app env.loop xs
```

```
let fp2 = fun l env →  
  let rec loop = Clo (fp3, {f=env.g; loop}) in  
  app loop l
```

```
let fp1 = fun f env → Clo (fp2, {g=f})
```

```
let map = Clo (fp1, ())
```

Translations

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Reading

Functions

All source functions
enumerated as constructors

```
type ('a, 'b) fn =  
  | Map1 : (('a, 'b) fn, ('a list, 'b list) fn) fn  
  | Map2 : ('a, 'b) fn → ('a list, 'b list) fn  
  | Loop : ('a, 'b) fn → ('a list, 'b list) fn
```

Translations

```
let rec apply : type a b. (a, b) fn → a → b =  
  fun fn x → match fn, x with  
  | Map1, f → Map2 f  
  | Map2 f, l → apply (Loop f) l  
  | Loop f, l →  
    match l with  
    | [] → []  
    | x :: xs → apply f x :: apply (Loop f) xs
```

```
let map = Map1
```

Free-variable environments
attached to constructors

Reading

Free variables passed as additional parameters

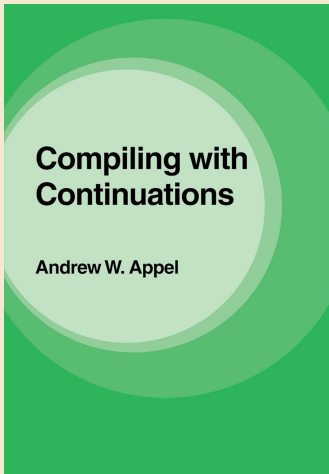
```
let map = fun f → fun l →
  let rec loop = fun l f →
    match l with
    | [] → []
    | x :: xs → f x :: loop xs f
  in loop l f
```

Functions moved to the top level

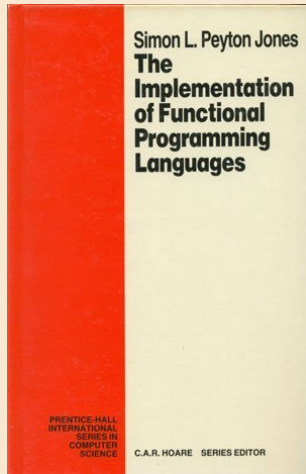
```
let rec loop = fun l f →
  match l with
  | [] → []
  | x :: xs → f x :: loop xs f
let map = fun f → fun l → loop l f
```

Reading

Functions



(for closure conversion)



(for lambda lifting)

Translations

Reading



Paper 1: *Defunctionalization at Work*

Functions

Defunctionalization at Work *

Olivier Danvy and Lasse R. Nielsen

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June, 2001

Abstract

Reynolds's defunctionalization technique is a whole-program transformation from higher-order to first-order functional programs. We study practical applications of this transformation and uncover new connections between seemingly unrelated higher-order and first-order specifications and between their correctness proofs. Defunctionalization therefore appears both as a springboard for revealing new connections and as a bridge for transferring existing results between the first-order world and the higher-order world.

“Defunctionalization is thus a whole-program transformation where function types are replaced by an enumeration of the function abstractions in this program.

[...]

Compared to closure conversion and to combinator conversion, defunctionalization has been used very little.

[...]

...one can use several apply functions, e.g., grouped by types, ...One can also defunctionalize a program selectively, e.g., only its continuations ...One can even envision a lightweight defunctionalization”

Reading



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Reading

We present choice conversion as an example of a type-directed and loss-preserving translation. In general, such

we use existential type abstraction to ensure the privacy of environment representation in much the same way that Pierce and Turner hide the representation types of instance variables”

Paper 3: *Lambda-Lifting in Quadratic Time*

Functions

Lambda-Lifting in Quadratic Time *

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June 17, 2004

Abstract

Lambda-lifting is a program transformation that is used in compilers, partial evaluators, and program transformers. In this article, we show how to reduce its complexity from cubic time to quadratic time, and we present a flow-sensitive lambda-lifter that also works in quadratic time.

Lambda-lifting transforms a block-structured program into a set of recursive equations, one for each local function in the source program. Each equation carries extra parameters to account for the free variables of the corresponding local function *and of all its callees*. It is the search for these extra parameters that yields the cubic factor in the traditional formulation of lambda-lifting, which is due to Johnsson. This search is carried out by computing a transitive closure.

To reduce the complexity of lambda-lifting, we partition the call graph of the source program into strongly connected components, based on the simple observation that *all functions in each component need the same extra parameters and thus a transitive closure is not needed*. We therefore simplify the search for extra parameters by treating each strongly connected component instead of each function as a unit, thereby reducing the time complexity of lambda-lifting from $O(n^3)$ to $O(n^2)$, where n is the size of the program.

Since a lambda-lifter can output programs of size $O(n^2)$, our algorithm is asymptotically optimal.

Keywords

Block structure, lexical scope, functional programming, inner classes in Java.

“Lambda-lifting is a program transformation that is used in compilers, partial evaluators, and program transformers.

[...]

Lambda-lifting transforms a block-structured program into a set of recursive equations, one for each local function in the source program. Each equation carries extra parameters to account for the free variables of the corresponding local function *and of all its callees*.

[...]

Since a lambda-lifter can output programs of size $O(n^2)$, our algorithm is asymptotically optimal.”

Translations

Reading



Functions

Relations

How are closure conversion and lambda lifting related?

How are closure conversion and defunctionalization related?

Preservation

Can the result of each translation always be given a type?

Do the translations extend to languages with more sophisticated type systems?

Variations

Can the translations be applied selectively? Do they support separate compilation?

Facilitation

What analyses and subsequent translations do the translations facilitate?

Reading

