Advanced topics in programming languages — Michaelmas 2025

Taming functions

Jeremy Yallop

jeremy.yallop@cl.cam.ac.uk

Translations

Functions are hard to handle





let rec loop = f x :: loop xs in loop l

can't know all possible functions in advance

analysis is difficult

nesting makes program transformation difficult

control flow is implicit/unclear

let map = fun f \rightarrow fun 1

Some taming translations

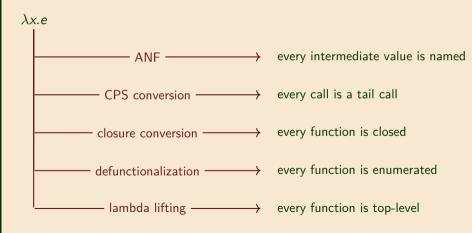




Translations

Reading

We'll consider total translations that introduce syntactic invariants:



How should we view these translations?





Translations

Translations are not optimizations

Translations don't make programs faster (and may well make them slower)

Translations don't use heuristics, and aren't sensitive to structure

Translations are representation changes

The translation targets are inconvenient as programming languages

Changing representation makes analysis and further translation easier

Translations by example

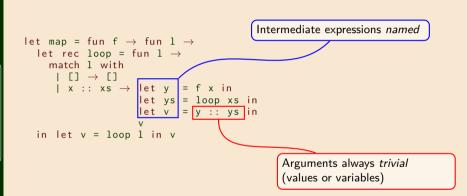
Translation to A-normal form

Functions

Translations

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Reading



Full details: The Essence of Compiling with Continuations (PLDI 1993)

Cormac Flanagan, Amr Sabry, Bruce F. Duba, Matthias Felleisen

Translation to continuating-passing style

Functions

Translations

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```
Computed values passed to continuations,
                                                                    Every function passed a continuation k,
    not returned
                                                                     the rest of the computation
let map_cps = fun/f k \rightarrow k (fun 1 k \rightarrow
   let rec loop = \begin{bmatrix} fun & 1 & k \end{bmatrix} \rightarrow \begin{bmatrix} fun & 1 & k \end{bmatrix}
     match 1 with
          [] \rightarrow k []
                           f x (fun y \rightarrow
                                   loop xs (fun vs -
                                                    (y :: ys))
  in loop 1 k)
let id x = x
let map = fun f 1 \rightarrow \text{map\_cps} (fun x k \rightarrow k (f x)) id 1 id
                               Every call a tail call
```

Closure conversion

Functions

Translations



Reading

```
All functions closed (i.e. C-style function
       pointers)
type ('a, 'b) clo =
   Clo : ('a \rightarrow 'env \rightarrow 'b) * 'env \rightarrow ('a, 'b) clo
let app (Clo (f. env)) x = f x env
let fp3 = fun 1 env \rightarrow match 1 with
      \Gamma \rightarrow \Gamma
     x :: xs \rightarrow app env.f x :: app env.loop xs
let fp2 = fun 1 env \rightarrow
   let rec loop = Clo (fp3, {f=env.g; loop}) in
   app loop 1
let fp1 = fun f env \rightarrow Clo (fp2, {g=f})
let map = Clo(fp1, ())
```

Free-variable environments bundled with functions

Defunctionalization

Functions

Translations



Reading

```
All source functions enumerated as constructors
```

```
 | \mbox{ Map2} : \mbox{ $('a, 'b)$ fn } \rightarrow \mbox{ $('a \mbox{ list, 'b list) fn } \mbox{} + \mbox{ $('a \mbox{ list, 'b list) fn } \mbox{} + \mb
```

Map1 : (('a, 'b) fn, ('a list, 'b list) fn) fn

type ('a, 'b) fn =

let map = Map1

Free-variable environments attached to constructors

Lambda lifting

Functions

Translations



```
Free variables passed as additional parameters

let map = fun f \rightarrow fun 1 \rightarrow let rec loop = fun 1 \rightarrow match 1 with | [] \rightarrow [] | x :: xs \rightarrow f x :: loop xs f in loop 1 f
```

```
Functions moved to the top level
```

```
let rec loop = fun l f \rightarrow match l with \mid [] \rightarrow [] \mid x :: xs \rightarrow f x :: loop xs f
```

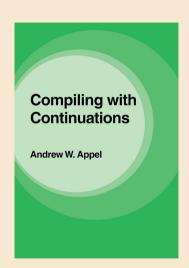
let map = fun f \rightarrow fun l \rightarrow loop l f

Background reading

Functions

Translations





(for closure conversion)



(for lambda lifting)

Paper 1: Defunctionalization at Work

Functions

Translation

Reading

Defunctionalization at Work *

Olivier Danvy and Lasse R. Nielsen

BRICS †

Department of Computer Science University of Aarhus [‡]

June, 2001

Abstract

Reynolds's defunctionalization technique is a whole-program transformation from higher-order to first-order functional programs. We study practical applications of this transformation and uncover new connections between seemingly unrelated higher-order and first-order specifications and between their correctness proofs. Defunctionalization therefore appears both as a springboard for revealing new connections and as a bridge for transferring existing results between the first-order world and the higher-order world.

"Defunctionalization is thus a wholeprogram transformation where function types are replaced by an enumeration of the function abstractions in this program.

[...]

Compared to closure conversion and to combinator conversion, defunctionalization has been used very little.

[...]

...one can use several apply functions, e.g., grouped by types, ...One can also defunctionalize a program selectively, e.g., only its continuations ...One can even envision a lightweight defunctionalization"

Paper 2: Typed closure conversion

Reading

Typed Closure Conversion'

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Clears, corrected is a greature transformation used by

compilers to operate code from data. Precious accounts

a a needs methodology for hallding compilers, because a

complex can use the tenes to implement efficient data can brian. Furthermers, translated translations facilitate se-

curity and debugging through automatic type decking, as

well as correct man a recomment a through the method of logical

a simple "cleaners as abjects" extended bighers ofer flowtions are viewed as abjects consisting of a single method

the rade; and a short instance variable, the environment is in the simple or pad case, the Pierre-Tumer smokel of also

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1 Intenduction

We present cleans convenies as a transferred and tennerowering translation for both the simple cond and

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Robert Harner 5 chief of Computer Science Carnegie Mellen University Parabomb, PA 82 Hel 911 reliteración a du

one ironment. The abstracted code is partially applied to an emilitie communication imment any idea the bindara ours rade and a more and attenue it is an iron ment. A critical decision in chosen conversion is the choice

the CAM-like [4], linked CAM-like [5], or beheld representssire to minimize cleaner creation time, the source communical representation is that the representation of the continuous is is good advantage to Shor and Appel [01] and Wand and

is not \$4. F. 3.13. M. This is advenue for complete that time. Hawever, when compiles trend tangeners, it is a few advantageous to necessarie tene information thereof each stage of the compiler, and is make use of types at links or even run time. Per example, Leray's representation analytions, and Obert's record completion [55] news a concession tion of tenes at con time to access community of a count. these proposed by Merrison et at [4] and Harper and Morriself [M], rely an analyzing types at run time to support callection [3, 37, 23] for both memoratic and notemorhear. To surpose one of these implementation strategies. it is necessary to propagate type information through the were conversion and into the convented cade. The number of this more is to demand to be have this can be done in both

a simple-treed and a pub mumble setting. We present the sure conversion as an example of a langdirected and inse-preserving immedation. In general, such "Closure conversion is a program transformation used by compilers to separate code from data.



translating to typed target languages is a useful methodology for building compilers



we use existential type abstraction to ensure the privacy of environment representation in much the same way that Pierce and Turner hide the representation types of instance variables"

Paper 3: Lambda-Lifting in Quadratic Time

Functions

Translations

Reading

Lambda-Lifting in Quadratic Time * Olivier Danvy Ulrik P. Schultz

 $\begin{array}{ccc} \mbox{Olivier Danvy} & \mbox{Ulrik P. Schultz} \\ \mbox{BRICS} \ ^{\dagger} & \mbox{ISIS Katrinebjerg} \\ \mbox{Department of Computer Science} \\ \mbox{University of Aarhus} \ ^{\ddagger} \end{array}$

June 17, 2004

Abstract

Lambda-lifting is a program transformation that is used in compilers, partial evaluators, and program transformers. In this article, we show how to reduce its complexity from cubic time to quadratic time, and we present a flow-sensitive lambda-lifter that also works in quadratic time.

Lambda-lifting transforms a block-structured program into a set of recursive equations, one for each local function in the source program. Each equation carries extra parameters to account for the free variables of the corresponding local function and of all its culliers. It is the search for these extra parameters that yields the cubic factor in the traditional formulation of lambda-lifting, which is due to Johnsson. This search is carried on the commuting a transitive domer.

To reduce the complexity of lambda-lifting, we partition the call graph of the source program into strongly connected components, based on the simple observation that all functions in each component need the same extra parameters and thus a trunsitive closure is not needed. We therefore instead procedure that the control of the contr

Since a lambda-lifter can output programs of size $O(n^2)$, our algorithm is asymptotically outimal

Keywords

Block structure, lexical scope, functional programming, inner classes in Java.

"Lambda-lifting is a program transformation that is used in compilers, partial evaluators, and program transformers.



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Writing suggestions

Relations

How are closure conversion and lambda lifting related?

How are closure conversion and defunctionalization related?

Preservation

Can the result of each translation always be given a type?

Can the translations be applied selectively? Do they support separate

Do the translations extend to languages with more sophisticated type systems?

Variations

compilation?

Facilitation

What analyses and subsequent translations do the translations facilitate?