Quantum Complexity Theory

Bell's inequality and non-local games

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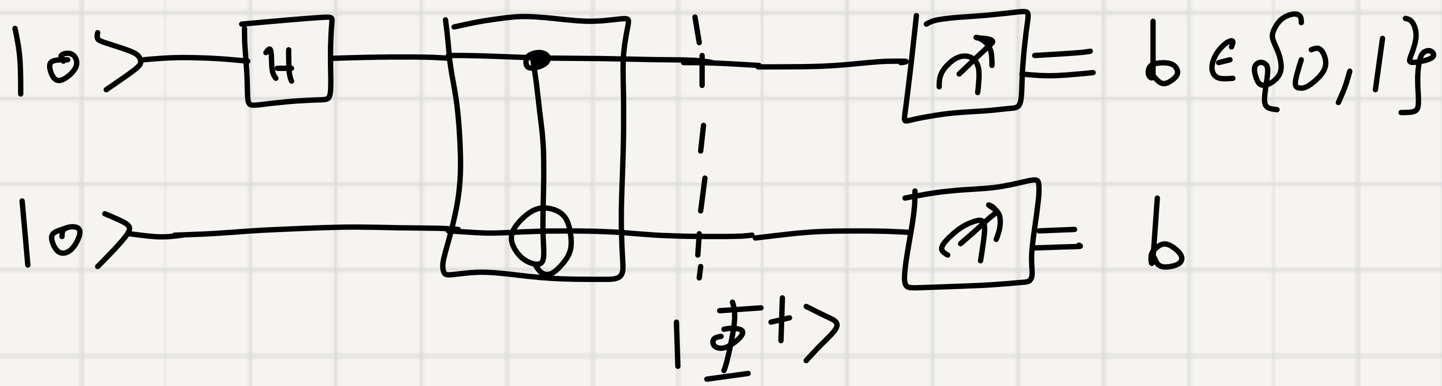
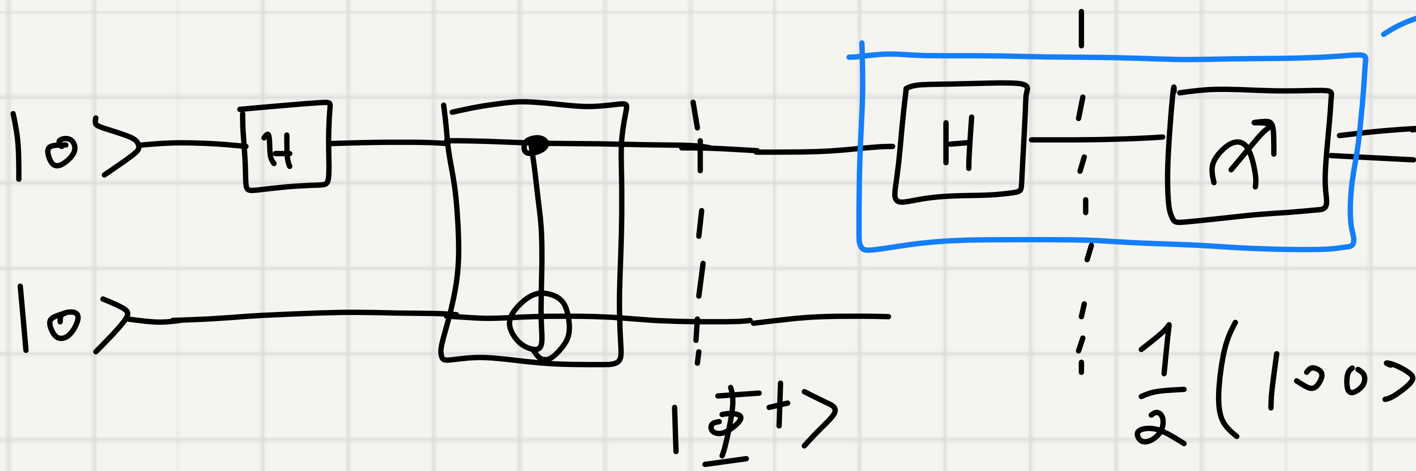
Einstein, Podolsky, Rosen 1935

FTL?!



$\rightarrow \{|-\rangle, |+\rangle\}$ measurement



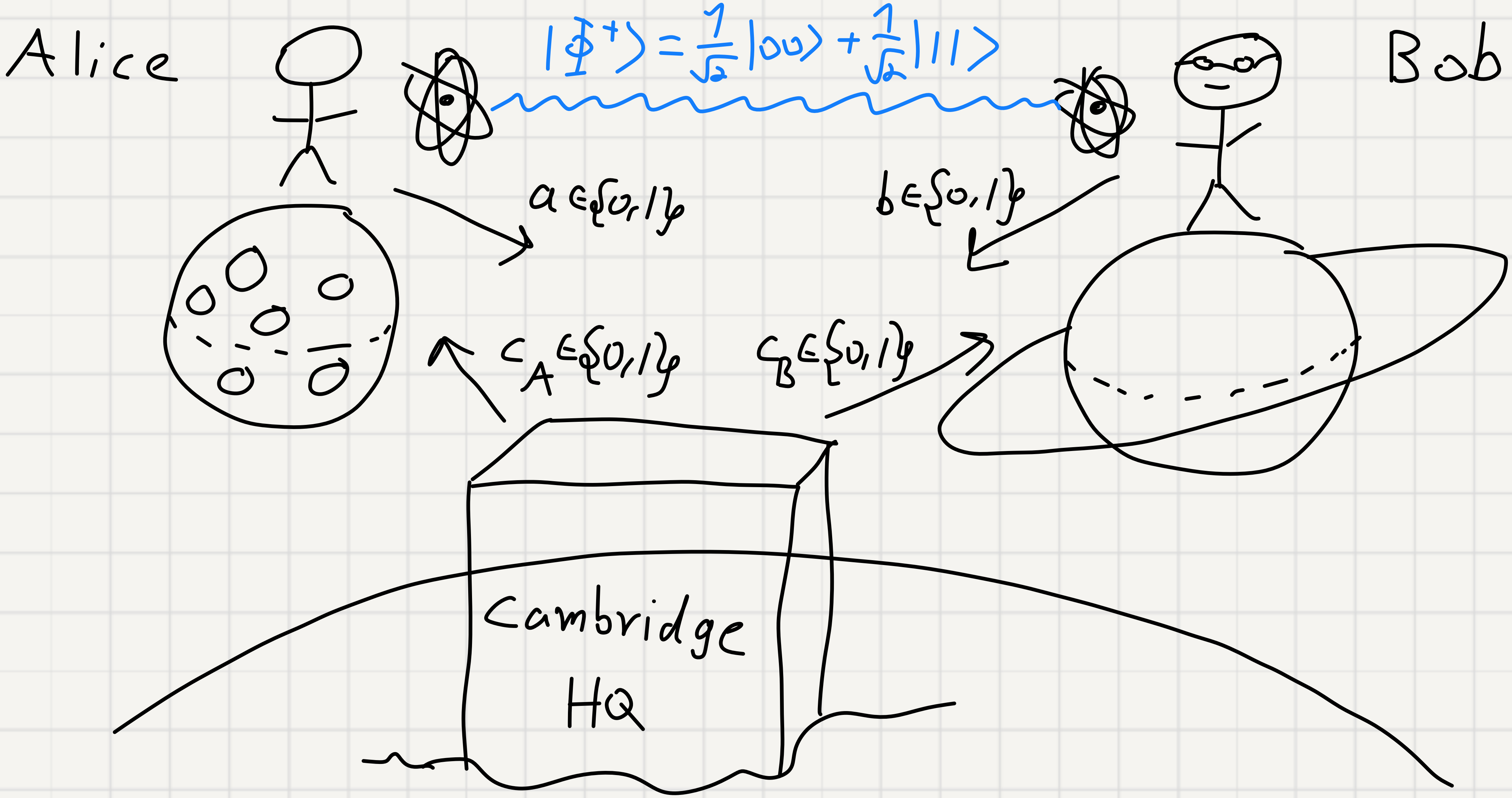
$\frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle - |11\rangle\right\}$

What is Bob's state?

- If Alice measured $|0\rangle$: $|0\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) = |0+\rangle$

- If Alice measured $|1\rangle$: $|1\rangle \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) = |1-\rangle$

Clauser, Horne, Shimony, Holt 1969 (CHSH game)



c_A, c_B chosen at random. Goal: If $c_A = c_B = 1$ $a \neq b$

o/w $a = b$

Def A <u>Bell inequality</u> of game G is an upper bound on the max succes prob. of a classical strategy

Claim $\text{Bell}(CHSH) < 0.75$

Proof By convexity, w.l.o.g., the strategy is deterministic.

Observe the game is won iff

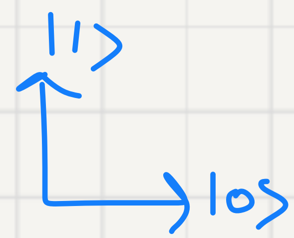
$$a(c_A) \oplus b(c_B) = c_A \cdot c_B$$

can only hold for 3/4 vals of c_A, c_B $\square$

Thm $\exists$ quantum strategy that wins CHSH w.p. ≥ 0.85

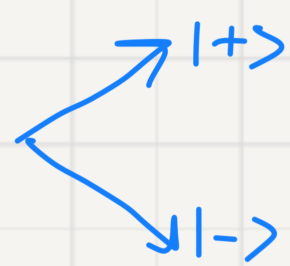
This was verified experimentally !

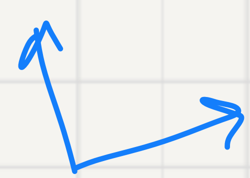
Proof consider the strategy:

Alice: If $c_A = 0$, measure 

send $a=0$ if $|0\rangle / |+\rangle$

$a=1$ o/w

If $c_A = 1$, measure 

Bob: If $c_B = 0$, measure  $|\pi/8\rangle = \cos\frac{\pi}{8}|0\rangle + \sin\frac{\pi}{8}|1\rangle$

If $c_B = 1$, measure  $|-\pi/8\rangle$

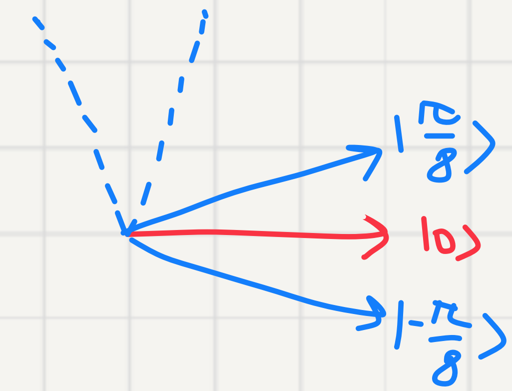
send $b=0$ if $|\pm\frac{\pi}{8}\rangle$

$b=1$ o/w

Analysis

Sps $c_A = 0$ and Alice measures $|0\rangle$. Then $a = 0$.

Bob's qubit collapsed to $|0\rangle$. Win iff $b = a = 0$.



$$\Pr\left[\left|\tfrac{\pi}{8}\right\rangle\right] = \Pr\left[\left|-\tfrac{\pi}{8}\right\rangle\right] = \cos^2\frac{\pi}{8} \geq 0.85$$

Same unless $c_A = c_B = 1$ (check!)

Then Alice measures in $\{|+\rangle, |-\rangle\}$. Sps $|-\rangle$. $a = 1$

Bob has $|-\rangle$. Win iff $b = 0$.

$$\Pr\left[\left|-\tfrac{\pi}{8}\right\rangle\right] = \cos^2\frac{\pi}{8} \geq 0.85$$

$\Rightarrow$ local realism is false $\square$

Discussion

- Magic square game
- Tsierlson bounds
- Hidden local variable vs FTL
- If we could clone, we could learn the qubit...
- Mixed states