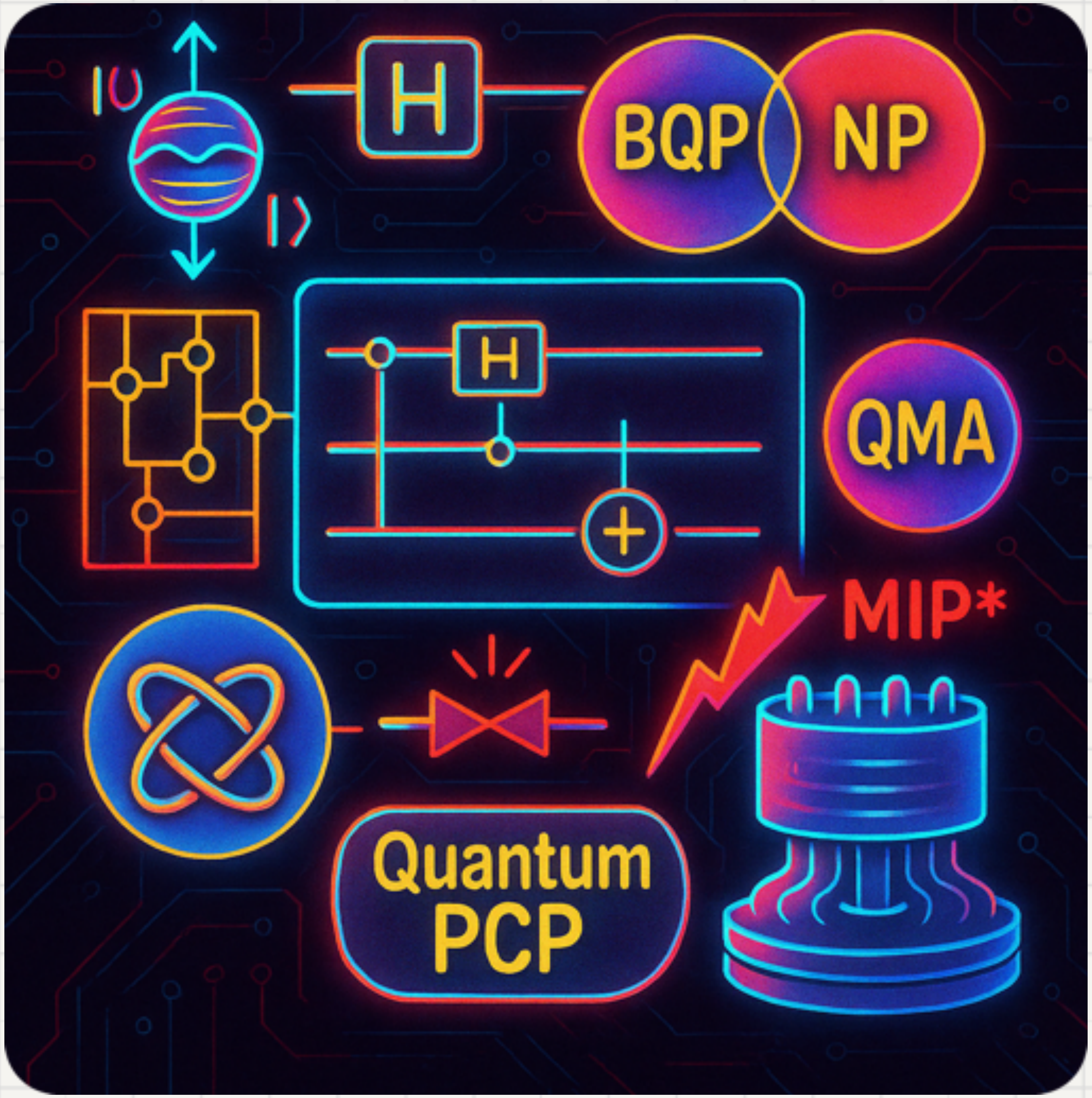
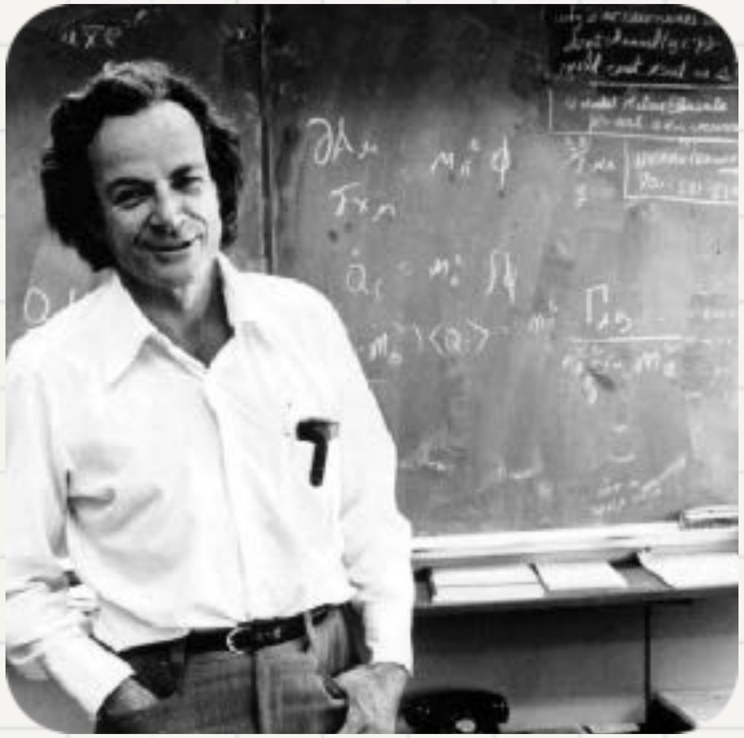
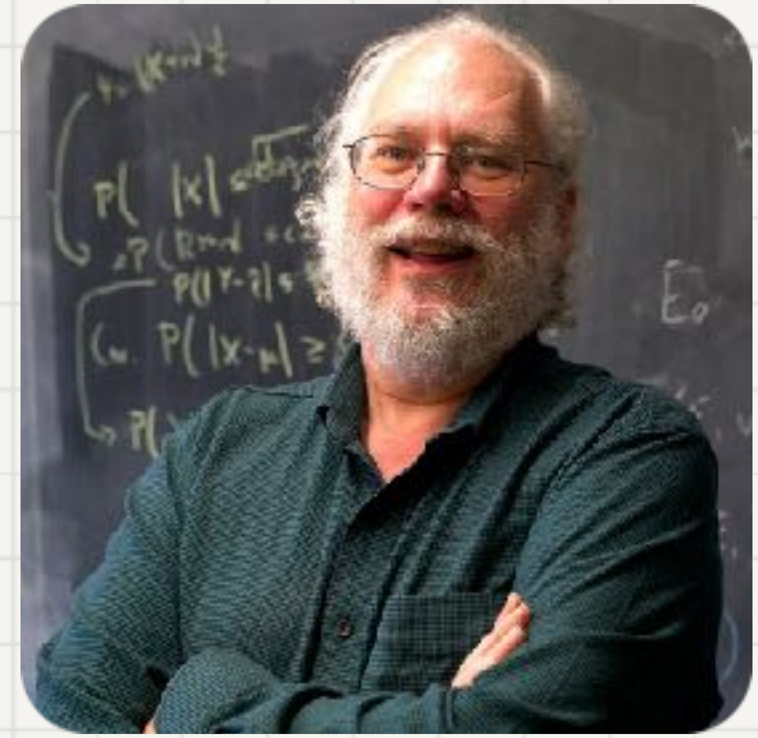
Quantum Complexity Theory

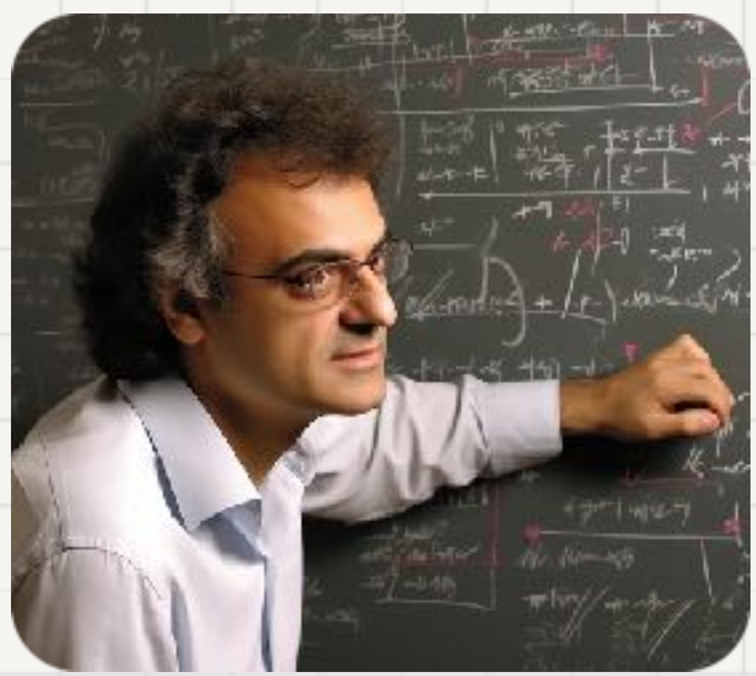


Tom Gur

A story via a quiz: name the scientists







Quantum Complexity Theory

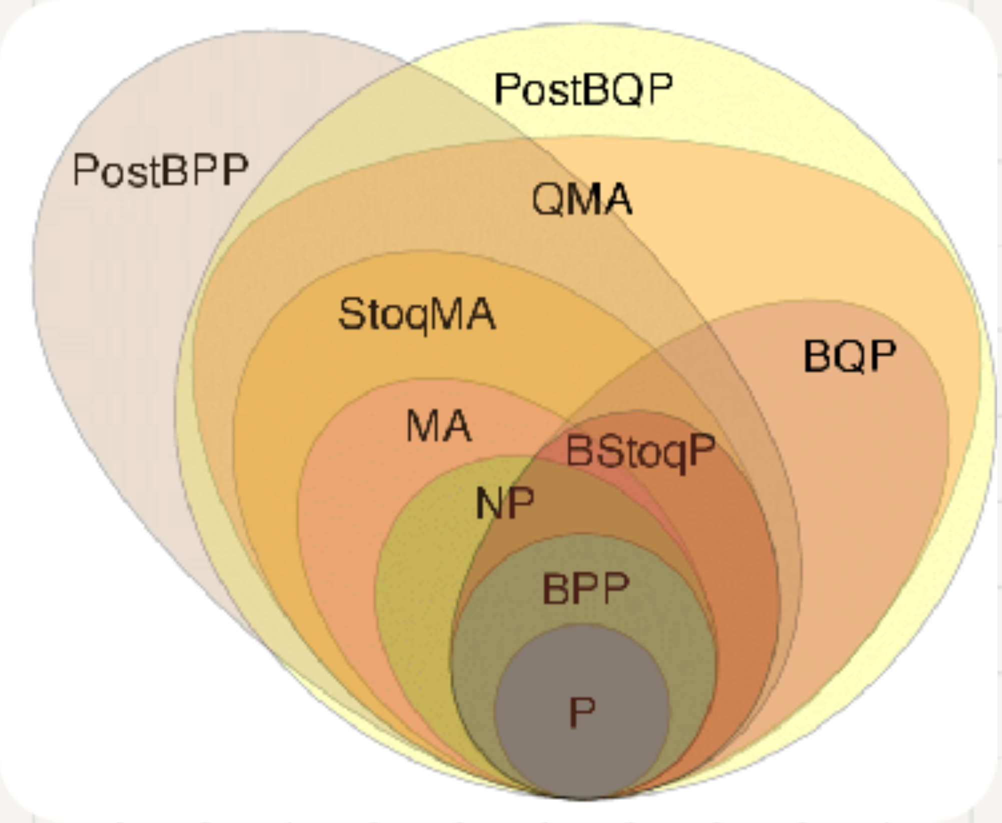
Goal: Understand the power and limitations of quantum computing

Why care?

Theory: Quantum computing as the "algebraic completion" of computing

Practice: Discover new quantum algorithms!

Interdisciplinary: Deep connections to physics and mathematics



"if you teach a course on quantum mechanics, and the students don't have nightmares for weeks, tear their hair out, wander around with bloodshot eyes, etc., then you probably didn't get the point across."

Time travel

Parallel universes

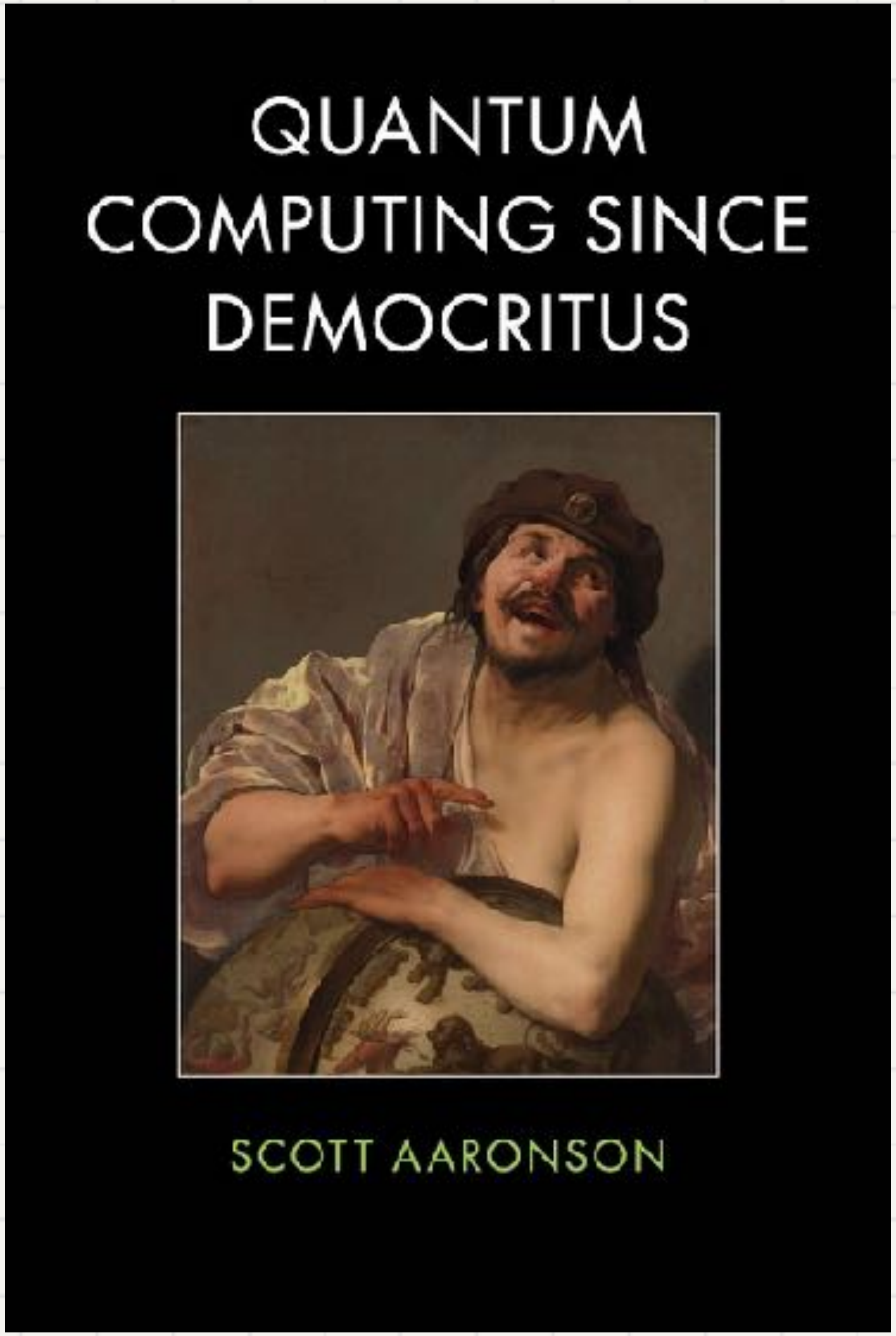
Teleportation

Cloning

Faster-than-light communication

Determinism

Chaos



Course structure

Phase 1 Review/crash-course on quantum computing

Supporting material:
* https://www.youtube.com/playlist?list=PLm3J0oaFux3YL5qLskC6xQ24JpMwOAeJz
* https://www.scottaaronson.com/qclec.pdf

Phase 2 Basics of quantum complexity theory

(Complementing Cambridge's Quantum Computing course)

Phase 3 Advanced topics: Quantum PCP, learning algorithms, zero-knowledge proofs, supremacy, lower bounds, and more...

The last phase will be covered in seminars

Assessment

Seminars (20%)

Lectures	Topic		
Lecture 1	Basics of Quantum Computing		
Lecture 2	BQP and QMA		
Lecture 3	Bell's inequality		
Lecture 4	Mixed states		
Lecture 5	Fourier Sampling		
Lecture 6	Hidden Subgroup Problem		
Seminars		**Presenter 1**	**Presenter 2**
Lecture 7	Quantum Cook-Levin		
Lecture 8	Quantum PCP and NLTS		
Lecture 9	Quantum Interactive Proofs		
Lecture 10	MIP* = RE		
Lecture 11	Quantum Zero-Knowledge Proofs		
Lecture 12	Quantum Supremacy: Yamakawa-Zhandry		
Lecture 13	Quantum Tomography		
Lecture 14	Bell sampling		
Lecture 15	Quantum limitations via the Polynomial method		
Lecture 16	QAC0 Lower bounds		

1) Choose a topic
2) Planning meeting
3) Review meeting
4) Presentation in pairs

Final project (80%)

Builds on the presentation and lectures

Read a paper; summarize it; a small research project (MPhil/Part III)

The plan (basics)

Lecture 1. Introduction and basics of Quantum Mechanics

Lecture 2. BQP and QMA

Lecture 3. Bell's inequality

Lecture 4. Mixed states

Lecture 5. Fourier sampling

Lecture 6. Hidden subgroup problem

The plan (topics)

Lecture 7. Quantum Cook-Levin

Lecture 8. Quantum PCP and NLTS

Lecture 9. Quantum Interactive Proofs

Lecture 10. MIP*=RE

Lecture 11. Quantum Zero-Knowledge Proofs

Lecture 12. Quantum Supremacy: Yamakawa-Zhandry

Lecture 13. Tomography and learning

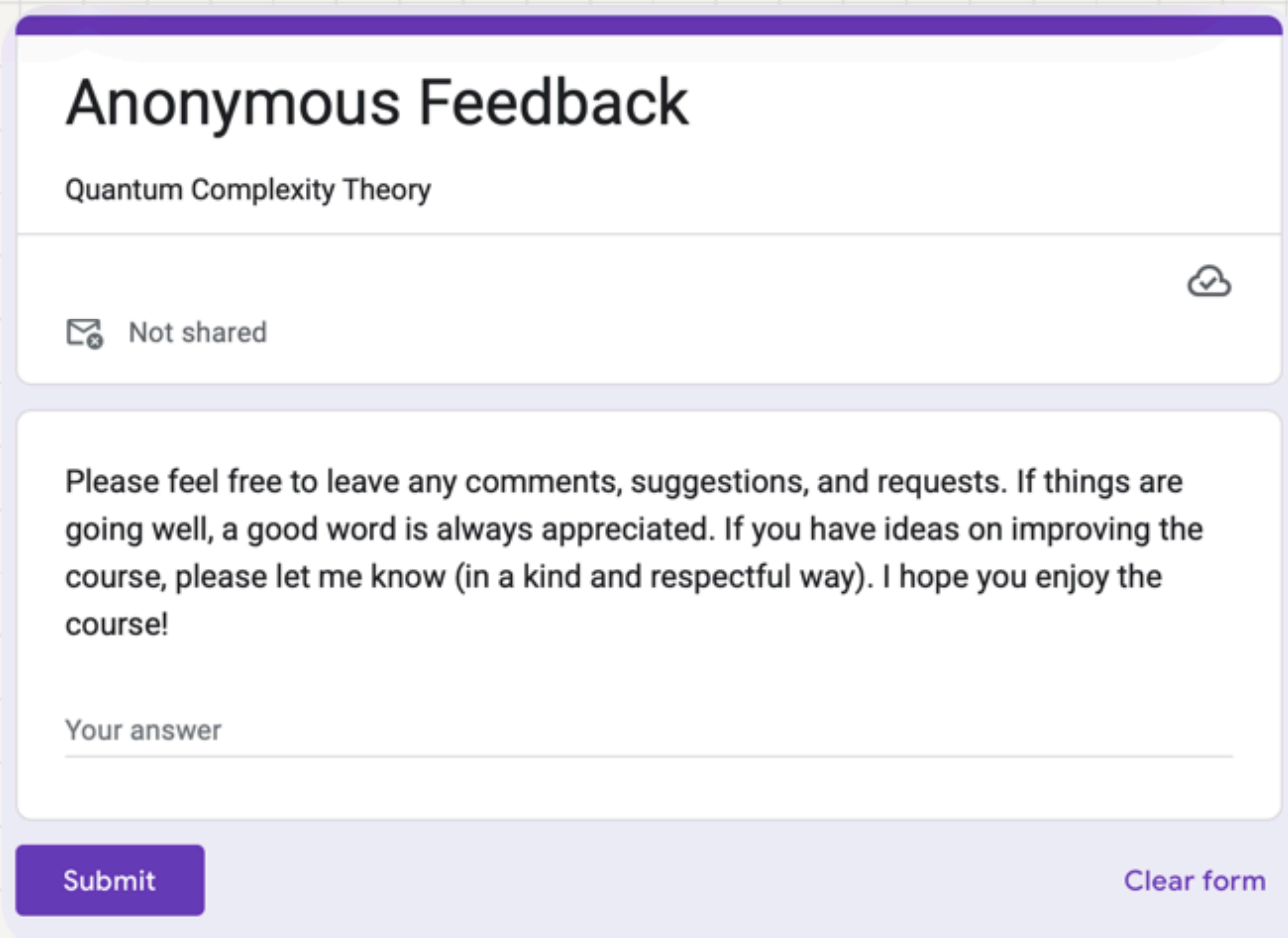
Lecture 14. Bell Sampling

Lecture 15. Polynomial method

Lecture 16. Shallow circuits and Pauli analysis

Anonymous feedback

A chance to make a difference in time for this term!



Anonymous Feedback

Quantum Complexity Theory

Not shared

Please feel free to leave any comments, suggestions, and requests. If things are going well, a good word is always appreciated. If you have ideas on improving the course, please let me know (in a kind and respectful way). I hope you enjoy the course!

Your answer

Submit

Clear form

https://www.cl.cam.ac.uk/teaching/2526/L330/materials.html

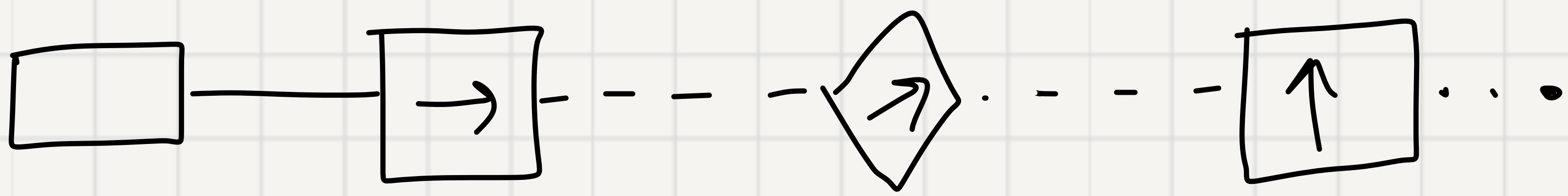
A crash course

on

Quantum Computing

Quantum Mechanics as non-commutative prob. theory

The 3 filter experiment



$1/8$

0

classical: $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}$

Quantum: $H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $D = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$ $V = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$$H \cdot V = 0 \neq H D V$$

Classical physics & probability theory

Consider a physical system S with d states $\{0,1,\dots,d-1\}$.

Let $s_0,\dots,s_{d-1} \geq 0$ s.t. $\Sigma s_i = 1$ a distribution.

modelling our knowledge

- represented by $\begin{pmatrix} s_0 \\ \vdots \\ s_{d-1} \end{pmatrix} \in \mathbb{R}^d$
- observing/measuring S gives i w.p. s_i. Then collapses.
- S evolves <u>stochastically</u> $s \mapsto A \cdot s$ mapping dist-to-dist

  $\forall a_{ij} \geq 0,\ \forall j\ \forall a_{ij} = 1$
- systems S and T combined are represented by $S \otimes T$.

Dirac notation

Consider $\mathbb{C}^d$. Denote $|i\rangle = e_i \quad \forall i \in [d]$ "ket"

$$|i\rangle = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \leftarrow i$$

col vector

$|\psi\rangle = \sum_i \alpha_i |i\rangle$

"bra"

$\langle\psi| = \left(|\psi\rangle\right)^* = \sum_i \alpha_i^* \langle i|$ → Hermitian conj row vector

- Inner product: $\langle\theta|\psi\rangle = \sum_i \alpha_i \beta_i^*$
- Outer product: $|\psi\rangle\langle\theta| = M$ s.t. $M_{ij} = \alpha_i \beta_j^*$
- $\forall M \quad M = \sum_{i,j} M_{ij} |i\rangle\langle j|$
- For $M = |\psi\rangle\langle\theta|$, $v = |\phi\rangle$: $\overbrace{|\psi\rangle\langle\theta|}^{M}\overbrace{|\phi\rangle}^{v} = \overbrace{|\psi\rangle}^{\text{Vec}} \cdot \overbrace{\langle\theta|\phi\rangle}^{\text{Scalar}}$

Quantum mechanics

Postulate 1 (Superposition) A quantum state is a

vector $|\psi\rangle \in \mathbb{C}^d$ s.t. $\| |\psi\rangle \|_2^2 = \langle \psi | \psi \rangle = \sum_{i=0}^{d-1} |\alpha_i|^2 = 1$

$$|\psi\rangle = \begin{pmatrix} \alpha_0 \\ \vdots \\ \alpha_{d-1} \end{pmatrix}$$

Hilbert space

Ex Qubit $\begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix} \in \mathbb{C}^2$ s.t. $|\alpha_0|^2 + |\alpha_1|^2 = 1$

amplitudes (vs probabilities)

Two "novelties": I complex numbers (negative!)

II ℓ^2 – norm

Postulate II (Measurement)

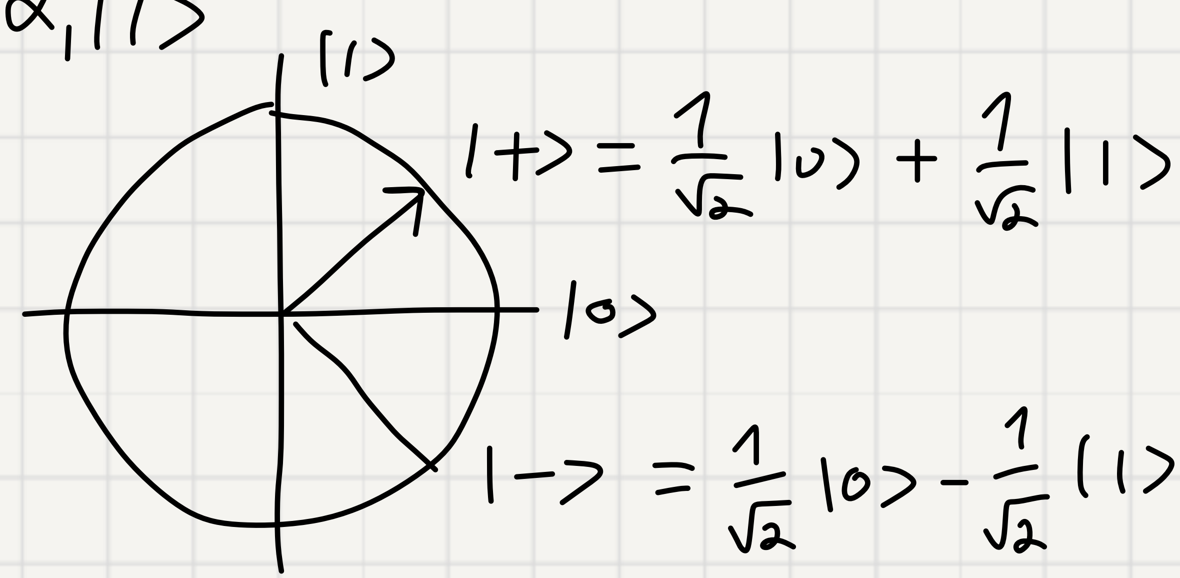
Measuring a state $|\psi\rangle = \sum \alpha_i |i\rangle = \begin{pmatrix} \alpha_0 \\ \vdots \\ \alpha_{d-1} \end{pmatrix}$ in ort. basis

$\{|b_0\rangle, \ldots, |b_{d-1}\rangle\}$ collapses it $|b_i\rangle$ w.p. $|\langle b_i|\psi\rangle|^2$.

<u>Ex (Born's rule)</u> Measuring $\begin{pmatrix} \alpha_0 \\ \vdots \\ \alpha_{d-1} \end{pmatrix}$ in the computational

basis $\{e_i\}$ collapses it to $|i\rangle$ w.p. $|\alpha_i|^2$

Qubit $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$



Postulate III (Unitary evolution)

Quantum states evolve via unitary transformations;

$|\psi\rangle \to U|\psi\rangle$, where $UU^{\dagger} = I$ $\quad (U^{-1} = U^{\dagger})$

- U maps unit vectors to unit vectors.
  (quantum states)

- U preserves inner products

$$\langle\psi|U^{\dagger}U|\theta\rangle = \langle\psi|U^{-1}U|\theta\rangle = \langle\psi|\theta\rangle$$

$$\underbrace{X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}}_{\text{Pauli}} \qquad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

Postulate IV (Entanglement)

The joint state of $|\psi\rangle$ and $|\theta\rangle$ is $|\psi\rangle \otimes |\theta\rangle$

Ex $\quad |0\rangle \otimes |0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad |0\rangle \otimes |1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \cdots$

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle \;, \quad |\theta\rangle = \beta_0 |0\rangle + \beta_1 |1\rangle$$

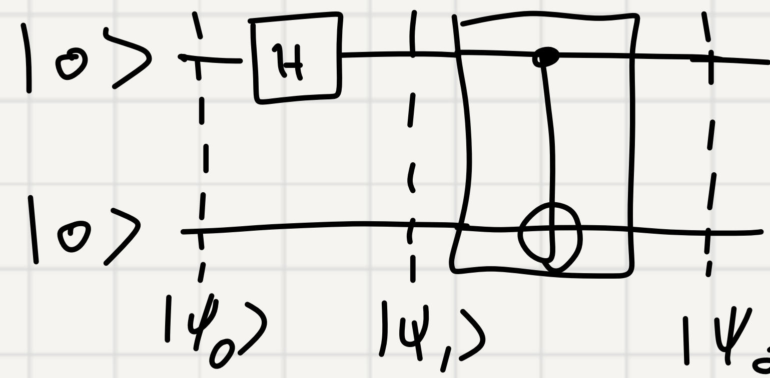
$$|\psi\rangle \otimes |\theta\rangle = \begin{pmatrix} \alpha_0 \beta_0 \\ \alpha_0 \beta_1 \\ \alpha_1 \beta_0 \\ \alpha_1 \beta_1 \end{pmatrix} = \alpha_0 \beta_0 |00\rangle + \alpha_0 \beta_1 |01\rangle + \ldots$$

shortcut

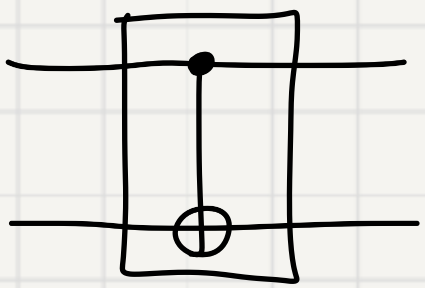
States that are not tensor products are entangled

Ex: $|EPR| = \frac{1}{\sqrt{2}} |00\rangle + \frac{1}{\sqrt{2}} |11\rangle$

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



$|\psi_0\rangle = |00\rangle$



$|00\rangle \to |00\rangle$
$|01\rangle \to |01\rangle$
$|10\rangle \to |11\rangle$
$|11\rangle \to |10\rangle$

$$|\psi_1\rangle = |+\rangle \otimes |0\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

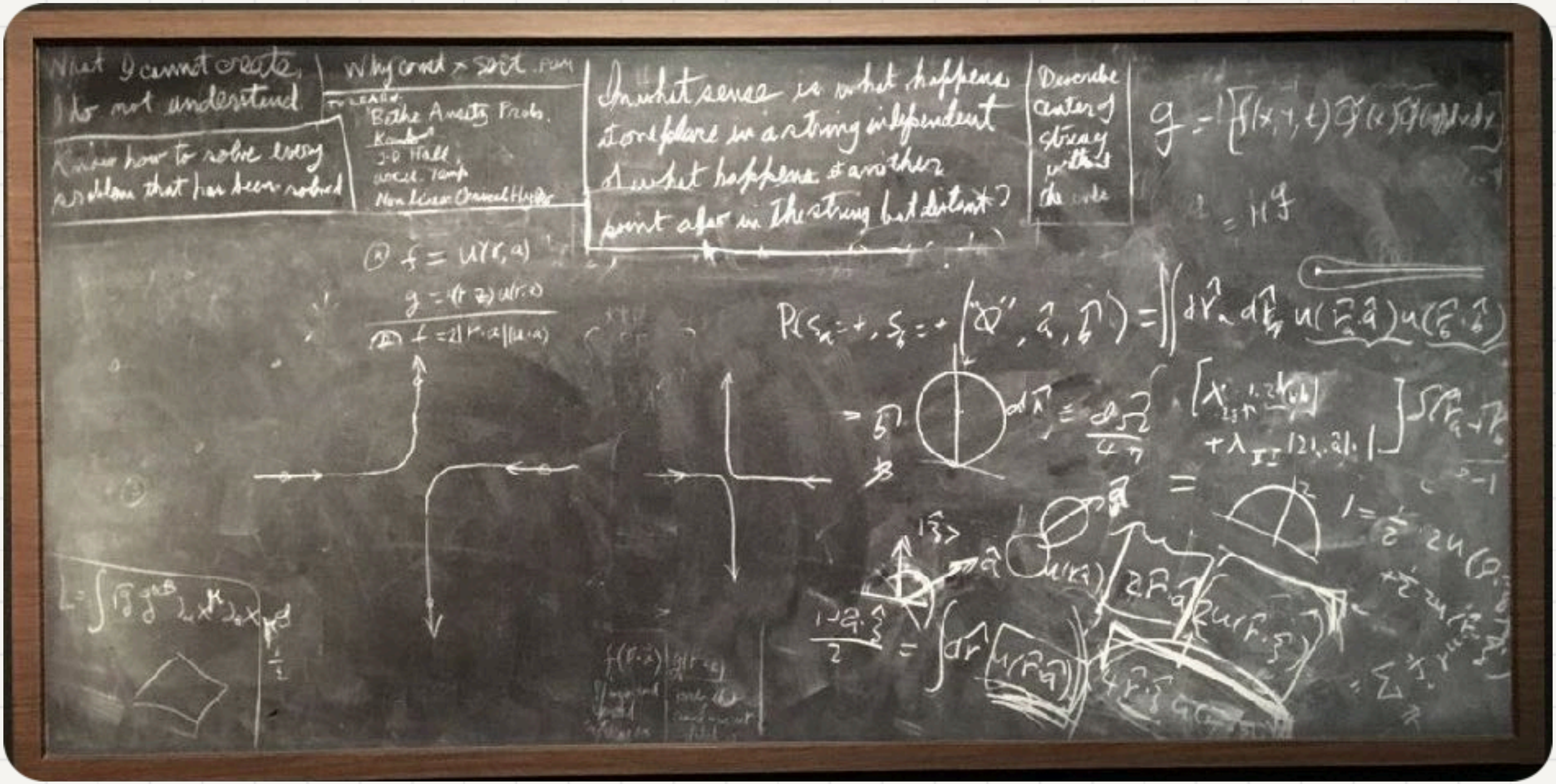
$$|\psi_2\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle = |EPR\rangle$$

Quantum mechanics is a linear theory

classical mechanics is non-linear

But QM is more general – how is that possible?

Quote of the day



What I cannot create, I do not understand — Richard Feynman